Velocity And Acceleration

- **Introduction**

Kinematics deals with study of relative motion between the various parts of the machines. Kinematics does not involve study of forces. Thus motion leads study of displacement, velocity and acceleration of a part of the machine.

Study of Motions of various parts of a machine is important for determining their velocities and accelerations at different moments.

As dynamic forces are a function of acceleration and acceleration is a function of velocities, study of velocity and acceleration will be useful in the design of mechanism of a machine. The mechanism will be represented by a line diagram which is known as configuration diagram. The analysis can be carried out both by graphical method as well as analytical method.

- **Some important Definitions**

Displacement: All particles of a body move in parallel planes and travel by same distance is known, linear displacement and is denoted by 'x'.

A body rotating about a fired point in such a way that all particular move in circular path angular displacement and is denoted by 'θ'.

Velocity: Rate of change of displacement is velocity. Velocity can be linear velocity of angular velocity.

Linear velocity is Rate of change of linear displacement= $V = \frac{dx}{dt}$

Angular velocity is Rate of change of angular displacement = $\omega = \frac{d\theta}{dt}$

Relation between linear velocity and angular velocity.

$$x = r\theta$$

$$\frac{dx}{dt} = r\frac{d\theta}{dt}$$

$$\mathbf{V = r\omega}$$

$$\omega = \frac{d\theta}{dt}$$

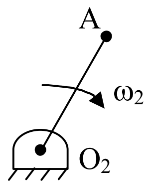
Acceleration: Rate of change of velocity

$f = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ Linear Acceleration (Rate of change of linear velocity)

Thirdly $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ Angular Acceleration (Rate of change of angular velocity)

We also have,

Absolute velocity: Velocity of a point with respect to a fixed point (zero velocity point).

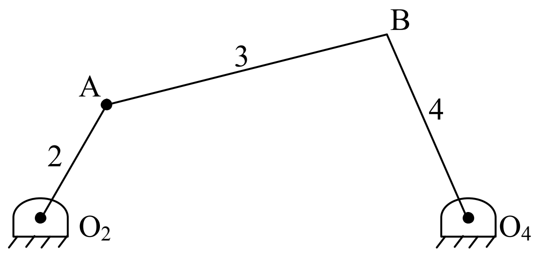


$V_a = \omega_2 \times r$

$V_a = \omega_2 \times O_2 A$

Ex: V_{ao_2} is absolute velocity.

Relative velocity: Velocity of a point with respect to another point 'x'

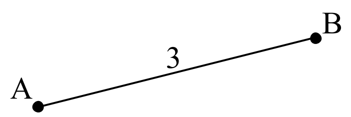


Ex: V_{ba} → Velocity of point B with respect to A

Note: Capital letters are used for configuration diagram. Small letters are used for velocity vector diagram.

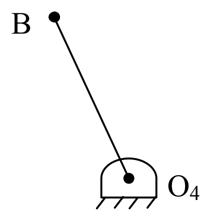
This is absolute velocity

∵ Velocity of point A with respect to O_2 fixed point, zero velocity point.

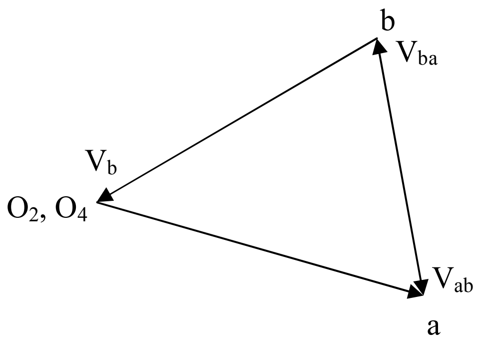


V_{ba} = or V_{ab}

V_{ba} = or V_{ab} Equal in magnitude but opposite in direction.



V_b → Absolute velocity is velocity of B with respect to O_4 (fixed point, zero velocity point)



Velocity vector diagram

Vector $\overrightarrow{O_2a}$ = V_a= Absolute velocity

Vector $\overrightarrow{ab}$ = V_{ab} } Relative velocity
$\overrightarrow{ba}$ = V_a

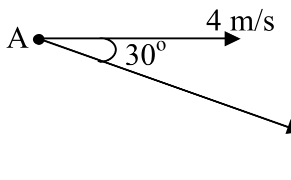
V_{ab} is equal magnitude with V_{ba} but is apposite in direction.

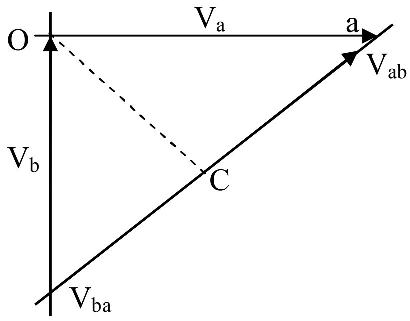
Vector $\overrightarrow{O_4\,b}$ = V_b absolute velocity.

To illustrate the difference between absolute velocity and relative velocity. Let, us consider a simple situation.

A link AB moving in a vertical plane such that the link is inclined at 30^o to the horizontal with point A is moving horizontally at 4 m/s and point B moving vertically upwards. Find velocity of B.

$V_a = 4$ m/s $\overrightarrow{ab}$	Absolute velocity	Horizontal direction (known in magnitude and directors)
$V_b = ?$ $\overrightarrow{ab}$	Absolute velocity	Vertical direction (known in directors only)





Velocity of B with respect to A is equal in magnitude to velocity of A with respect to B but opposite in direction.

- **Relative Velocity Equation**

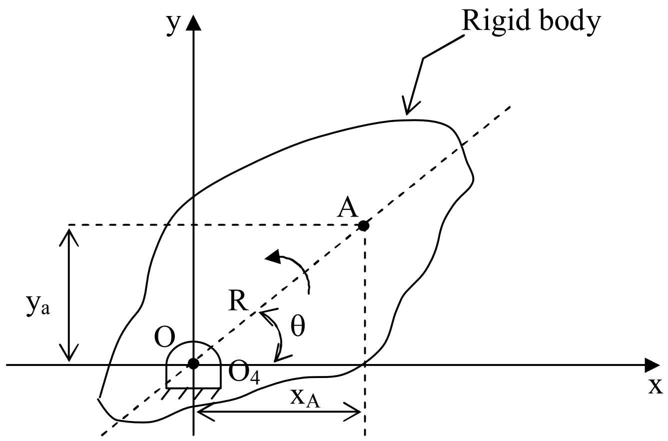


Fig. 1 Point O is fixed and End A is a point on rigid body.

Rotation of a rigid link about a fixed centre.

Consider rigid link rotating about a fixed centre O, as shown in figure. The distance between O and A is R and OA makes and angle 'θ' with x-axis next link

$x_A = R \cos \theta$, $y_A = R \sin \theta$.

Differentiating x_A with respect to time gives velocity.

$$\frac{d_{xA}}{dt} = R\left(-\sin\theta\right)\frac{d\theta}{dt}$$

$$= - R\omega \sin\theta$$

Similarly, $$\frac{dy_A}{dt} = R\left(-\cos\theta\right)\frac{d\theta}{dt}$$

$$= - R\omega \cos\theta$$

Let, $\frac{d_{xA}}{dt} = V_A^x$ $\qquad$ $\frac{d_{yA}}{dt} = V_A^y$

$$\omega = \frac{d\theta}{dt} = \text{angular velocity of OA}$$

$\therefore V_A^x = - R\omega \sin\theta$

$V_A^y = - R\omega \cos\theta$

$\therefore$ Total velocity of point A is given by

$$V_A = \sqrt{\left(-R\omega\sin\theta\right)^2 + \left(-R\omega\cos\theta\right)^2}$$

$$\mathbf{V_A = R\omega}$$

- **Relative Velocity Equation of Two Points on a Rigid link**

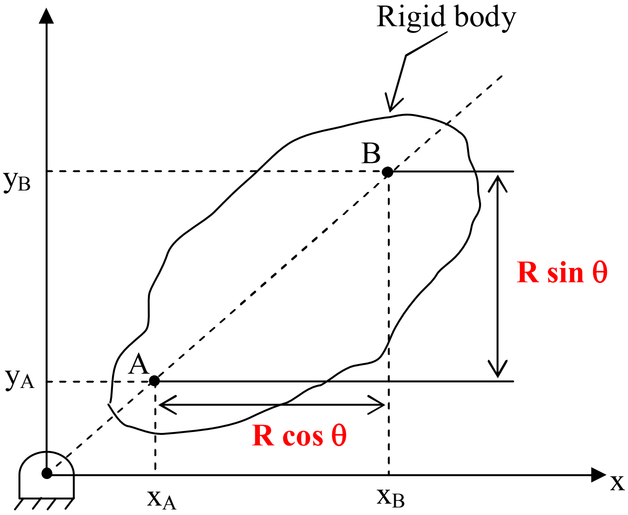


Fig. 2 Points A and B are located on rigid body

From Fig. 2

$$x_B = x_A + R\cos\theta \qquad y_B = y_A + R\sin\theta$$

Differentiating x_B and y_B with respect to time

we get,

$$\frac{d_{xB}}{dt} = V_B^x = \frac{d_{xA}}{dt} + R(-\sin\theta)\frac{d\theta}{dt}$$

$$= \frac{d_{xA}}{dt} + R\omega\sin\theta = V_A^x - R\omega\sin\theta$$

Similarly, $$\frac{d_{yB}}{dt} = V_B^y = \frac{d_{yA}}{dt} + R(\cos\theta)\frac{d\theta}{dt}$$

$$= \frac{d_{yA}}{dt} + R\omega\cos\theta = V_A^y - R\omega\cos\theta$$

$V_A = V_A^x \nrightarrow V_A^y$ = Total velocity of point A

Similarly, $V_B = V_B^x \nrightarrow V_B^y$ = Total velocity of point B

$= V_A^x \nrightarrow (R\omega\sin\theta) \nrightarrow V_A^y \nrightarrow R\omega\cos\theta$

$= (V_A^x \nrightarrow V_A^y) \nrightarrow (R\omega\sin\theta + R\,\omega\cos\theta)$

$= (V_A^x \;\overrightarrow{+}\; V_A^y)$ V_A Similarly, (R ωsin θ + Rω cos θ) = Rω

$\therefore V_B = V_A \overrightarrow{+} R\omega = V_A \overrightarrow{+} V_{BA}$

$\therefore \mathbf{V_{BA} = V_B - V_A}$

Velocity analysis of any mechanism can be carried out by various methods.

1. By graphical method
2. By relative velocity method
3. By instantaneous method

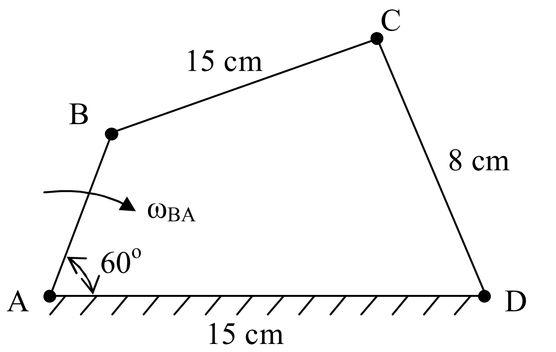
- **By Graphical Method**

The following points are to be considered while solving problems by this method.

1. Draw the configuration design to a suitable scale.
2. Locate all fixed point in a mechanism as a common point in velocity diagram.
3. Choose a suitable scale for the vector diagram velocity.
4. The velocity vector of each rotating link is $\perp^r$ to the link.
5. Velocity of each link in mechanism has both magnitude and direction. Start from a point whose magnitude and direction is known.
6. The points of the velocity diagram are indicated by small letters.

To explain the method let us take a few specific examples.

1. *Four – Bar Mechanism*: In a four bar chain ABCD link AD is fixed and in 15 cm long. The crank AB is 4 cm long rotates at 180 rpm (cw) while link CD rotates about D is 8 cm long BC = AD and $\underline{|BAD} = 60^o$. Find angular velocity of link CD.



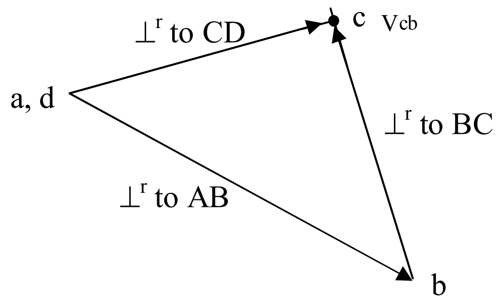
Configuration Diagram

Velocity vector diagram

$$V_b = \omega r = \omega_{ba} \text{ x } AB = \frac{2\pi x 120}{60} x 4 = 50.24 \text{ cm/sec}$$

Choose a suitable scale

$$1 \text{ cm} = 20 \text{ m/s} = \overrightarrow{ab}$$



$$V_{cb} = \overrightarrow{bc}$$

$$V_c = \overrightarrow{dc} = 38 \text{ cm/sec} = V_{cd}$$

We know that V =ω R

$$V_{cd} = \omega_{CD} \text{ x } CD$$

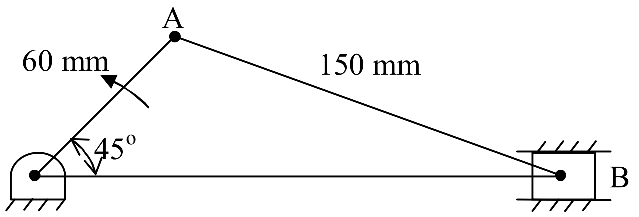
$$\omega_{cD} = \frac{V_{cd}}{CD} = \frac{38}{8} = 4.75 \text{ rad/sec (cw)}$$

2. *Slider Crank Mechanism*:

In a crank and slotted lever mechanism crank rotates of 300 rpm in a counter clockwise direction. Find

(i) Angular velocity of connecting rod and

(ii) Velocity of slider.



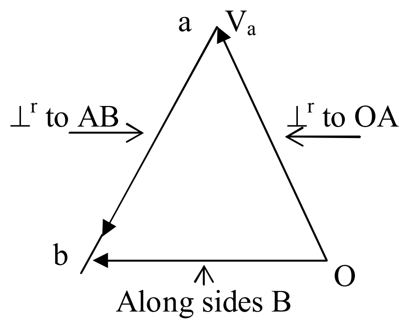
Configuration diagram

Step 1: Determine the magnitude and velocity of point A with respect to 0,

$$V_A = \omega_{O1A} \times O_2A = \frac{2\pi \times 300}{60} \times 60$$

$$= 600\ \pi \text{ mm/sec}$$

Step 2: Choose a suitable scale to draw velocity vector diagram.



Velocity vector diagram

$V_{ab} = \overrightarrow{ab} = 1300\text{mm/sec}$

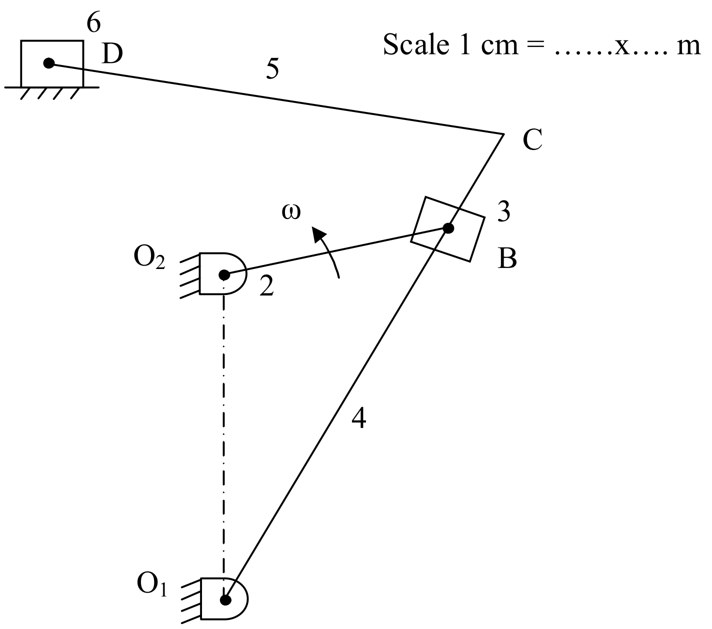
$$\omega_{ba} = \frac{V_{ba}}{BA} = \frac{1300}{150} = 8.66 \text{ rad/sec}$$

$V_b = \overrightarrow{ob}$ velocity of slider

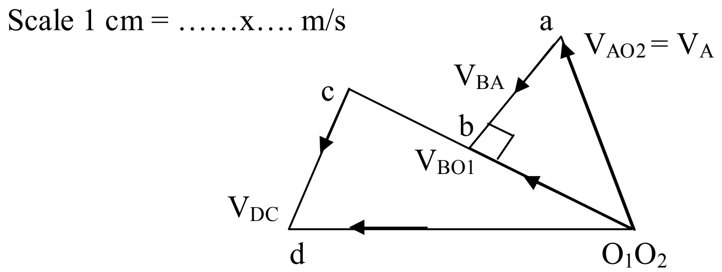
Note: Velocity of slider is along the line of sliding.

3. Shaper Mechanism:

In a crank and slotted lever mechanisms crank O_2A rotates at ω rad/sec in CCW direction. Determine the velocity of slider.



Configuration diagram



Velocity vector diagram

$V_a = \omega_2 \text{ x } O_2A$

$$\frac{\overrightarrow{O_1b}}{O_1B} = \frac{\overrightarrow{O_1c}}{O_1C}$$

To locate point C

$$\therefore \overrightarrow{O_1c} = \overrightarrow{O_1b}\left(\frac{O_1C}{O_1B}\right)$$

To Determine Velocity of Rubbing

Two links of a mechanism having turning point will be connected by pins. When the links are motion they rub against pin surface. The velocity of rubbing of pins depends on the angular velocity of links relative to each other as well as direction.

For example: In a four bar mechanism we have pins at points A, B, C and D.

$\therefore$ $V_{ra} = \omega_{ab}$ x ratios of pin A (r_{pa})

+ sign is used $\because$ ω_{ab} is CW and W_{bc} is CCW i.e. when angular velocities are in opposite directions use + sign when angular velocities are in some directions use -ve sign.

$\mathbf{V_{rb} = (\omega_{ab} + \omega_{bc})}$ **radius** $\mathbf{r_{pb}}$

$\mathbf{V_{rC} = (\omega_{bc} + \omega_{cd})}$ **radius** $\mathbf{r_{pc}}$

$\mathbf{V_{rD} = \omega_{cd}\ r_{pd}}$

Problems on velocity by velocity vector method (Graphical solutions)

Problem 1:

In a four bar mechanism, the dimensions of the links are as given below:

AB = 50 mm, BC = 66 mm

CD = 56 mm and AD = 100 mm

At a given instant when $\underline{|DAB} = 60^\circ$ the angular velocity of link AB is 10.5 rad/sec in CCW direction.

Determine,

i) Velocity of point C

ii) Velocity of point E on link BC when BE = 40 mm

iii) The angular velocity of link BC and CD

iv) The velocity of an offset point F on link BC, if BF = 45 mm, CF = 30 mm and BCF is read clockwise.

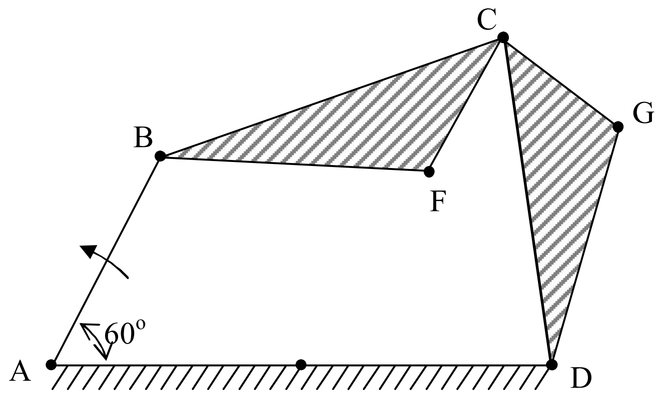
v) The velocity of an offset point G on link CD, if CG = 24 mm, DG = 44 mm and DCG is read clockwise.

vi) The velocity of rubbing of pins A, B, C and D. The ratio of the pins are 30 mm, 40 mm, 25 mm and 35 mm respectively.

Solution:

Step -1: Construct the configuration diagram selecting a suitable scale.

Scale: 1 cm = 20 mm



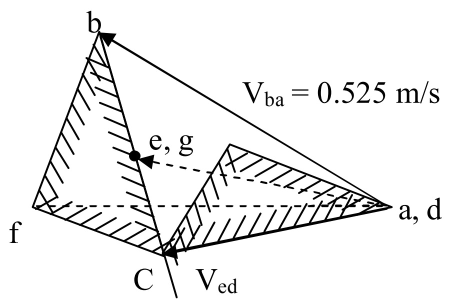
Step – 2: Given the angular velocity of link AB and its direction of rotation determine velocity of point with respect to A (A is fixed hence, it is zero velocity point).

$V_{ba} = \omega_{BA} \times BA$

$= 10.5 \times 0.05 = 0.525$ m/s

Step – 3: To draw velocity vector diagram choose a suitable scale, say 1 cm = 0.2 m/s.

- First locate zero velocity points.
- Draw a line $\perp^r$ to link AB in the direction of rotation of link AB (CCW) equal to 0.525 m/s.



- From b draw a line $\perp^r$ to BC and from d. Draw d line $\perp^r$ to CD to interest at C.
- V_{cb} is given vector bc $V_{bc} = 0.44$ m/s
- V_{cd} is given vector dc $V_{cd} = 0.39$ m/s

Step – 4: To determine velocity of point E (Absolute velocity) on link BC, first locate the position of point E on velocity vector diagram. This can be done by taking corresponding ratios of lengths of links to vector distance i.e.

$$\frac{be}{bc} = \frac{BE}{BC}$$

$$\therefore\ be = \frac{BE}{BC} \text{ x } V_{cb} = \frac{0.04}{0.066} \text{ x } 0.44 = 0.24 \text{ m/s}$$

Join e on velocity vector diagram to zero velocity points a, d / vector $\overrightarrow{de} = V_e = 0.415$ m/s.

Step 5: To determine angular velocity of links BC and CD, we know V_{bc} and V_{cd}.

$$\therefore\ V_{bc} = \omega_{BC} \text{ x } BC$$

$$\therefore\ \omega_{BC} = \frac{V_{bc}}{BC} = \frac{0.44}{0.066} = 6.6\ r/s\ .\ (cw)$$

Similarly, $V_{cd} = \omega_{CD} \text{ x } CD$

$$\therefore\ \omega_{CD} = \frac{V_{cd}}{CD} = \frac{0.39}{0.056} = 6.96\ \text{r/s}\ \ (CCW)$$

Step – 6: To determine velocity of an offset point F

- Draw a line $\perp^r$ to CF from C on velocity vector diagram.
- Draw a line $\perp^r$ to BF from b on velocity vector diagram to intersect the previously drawn line at 'f'.
- From the point f to zero velocity point a, d and measure vector fa to get

  $V_f = 0.495$ m/s.

Step – 7: To determine velocity of an offset point.

- Draw a line $\perp^r$ to GC from C on velocity vector diagram.

- Draw a line $\perp^r$ to DG from d on velocity vector diagram to intersect previously drawn line at g.
- Measure vector dg to get velocity of point G.

$$V_g = \overrightarrow{dg} = 0.305 \text{ m/s}$$

Step – 8: To determine rubbing velocity at pins

- Rubbing velocity at pin A will be

  $V_{pa} = \omega_{ab}$ x r of pin A

  $V_{pa} = 10.5 \times 0.03 = 0.315$ m/s

- Rubbing velocity at pin B will be

  $V_{pb} = (\omega_{ab} + \omega_{cb}) \times r_{pb}$ of point at B.

  [ω_{ab} CCW and ω_{cb}CW]

  $V_{pb} = (10.5 + 6.6) \times 0.04 = 0.684$ m/s.

- Rubbing velocity at point C will be

  $= 6.96 \times 0.035 = 0.244$ m/s

Problem 2:

In a slider crank mechanism the crank is 200 mm long and rotates at 40 rad/sec in a CCW direction. The length of the connecting rod is 800 mm. When the crank turns through 60^o from Inner-dead centre.

Determine,

i) The velocity of the slider

ii) Velocity of point E located at a distance of 200 mm on the connecting rod extended.

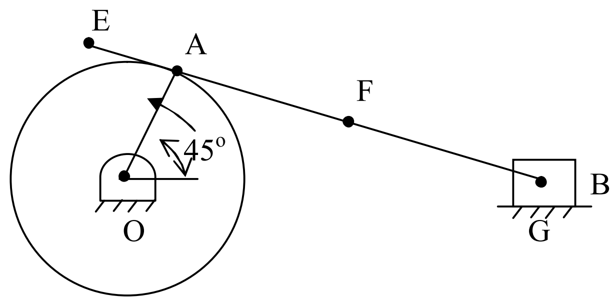
iii) The position and velocity of point F on the connecting rod having the least absolute velocity.

iv) The angular velocity of connecting rod.

v) The velocity of rubbing of pins of crank shaft, crank and cross head having pins diameters 80,60 and 100 mm respectively.

Solution:

Step 1: Draw the configuration diagram by selecting a suitable scale.



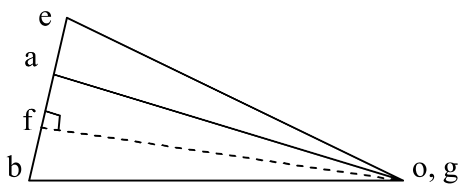
$V_a = W_{oa} \times OA$

$V_a = 40 \times 0.2$

$V_a = 8$ m/s

<u>Step 2</u>: Choose a suitable scale for velocity vector diagram and draw the velocity vector diagram.

- Mark zero velocity point o, g.
- Draw $\vec{oa}$ $\perp^r$ to link OA equal to 8 m/s



Scale: 1 cm = 2 m/s

- From a draw a line $\perp^r$ to AB and from o, g draw a horizontal line (representing the line of motion of slider B) to intersect the previously drawn line at b.
- $\vec{ab}$ give V_{ba}=4.8 m/sec

<u>Step – 3</u>: To mark point 'e' since 'E' is on the extension of link AB drawn $\vec{be} = \frac{BE}{AB} \times \vec{ab}$ mark the point e on extension of vector ba. Join e to o, g. $\vec{ge}$ will give velocity of point E.

$V_e = \vec{ge}$=8.4 m/sec

Step 4: To mark point F on link AB such that this has least velocity (absolute).

Draw a line $\perp^r$ to $\overrightarrow{ab}$ passing through o, g to cut the vector ab at f. From f to o, g. $\overrightarrow{gf}$ will have the least absolute velocity.

- To mark the position of F on link AB.

  Find BF by using the relation.

$$\frac{\overrightarrow{fb}}{BF} = \frac{\overrightarrow{ab}}{AB}$$

$$\mathbf{BF} = \frac{\overrightarrow{\mathbf{fb}}}{\mathbf{ab}} \;\mathbf{x}\; \mathbf{AB} = 200\text{mm}$$

Step – 5: To determine the angular velocity of connecting rod.

We know that $V_{ab} = \omega_{ab}$ x AB

$$\therefore\ \boldsymbol{\omega_{ab}} = \frac{\mathbf{V_{ab}}}{\mathbf{AB}} = \mathbf{6\ rad/sec}$$

Step – 6: To determine velocity of rubbing of pins.

- $V_{pcrankshaft} = \omega_{ao}$ x radius of crankshaft pin

  = 8 x 0.08

  = 0.64 m/s

- $V_{Pcrank\ pin} = (\omega_{ab} + \omega_{oa})\ r_{crank\ pin}$= (6 +8)0.06 =0.84 m/sec
- $V_{P\ cross\ head} = \omega_{ab}$ x $r_{cross\ head}$ = 6 x 0.1 = 0.6 m/sec

- **Problem 3:** A quick return mechanism of crank and slotted lever type shaping machine is shown in Fig. the dimensions of various links are as follows.

O_1O_2 = 800 mm, O_1B = 300 mm, O_2D = 1300 mm and DR = 400 mm

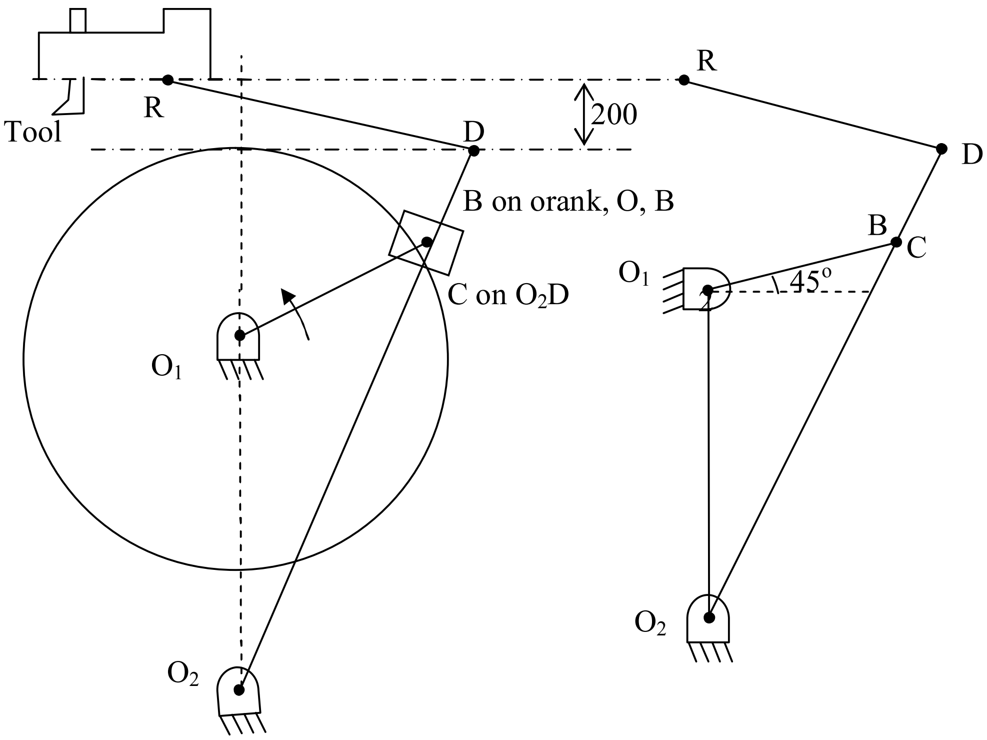
The crank O_1B makes an angle of 45^o with the vertical and relates at 40 rpm in the CCW direction. Find:

i) Velocity of the Ram R, velocity of cutting tool, and

ii) Angular velocity of link O_2D.

- **Solution:**

Step 1: Draw the configuration diagram.



Step 2: Determine velocity of point B.

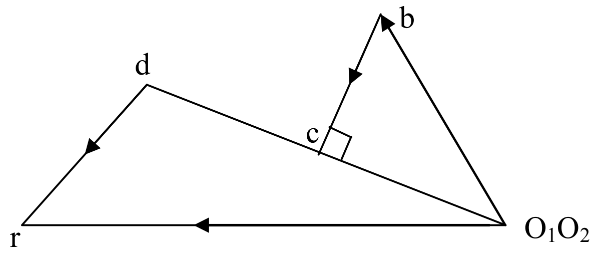
$V_b = \omega_{O1B} \times O_1B$

$\omega_{O1B} = \frac{2\pi N_{O1B}}{60} = \frac{2\pi \times 40}{60} = 4.18 \, rad/sec$

$V_b = 4.18 \times 0.3 = 1.254$ m/sec

Step 3: Draw velocity vector diagram.

Choose a suitable scale 1 cm = 0.3 m/sec



- Draw O_1b $\perp^r$ to link O_1B equal to 1.254 m/s.
- From b draw a line along the line of O_2B and from O_1O_2 draw a line $\perp^r$ to O_2B. This intersects at c $\overrightarrow{bc}$ will measure velocity of sliding of slider and $\overrightarrow{O_2C}$ will measure the velocity of C on link O_2C.
- Since point D is on the extension of link O_2C measure $\overrightarrow{O_2d}$ such that

$$\overrightarrow{O_2d} = \overrightarrow{O_2C}\ \frac{O_2D}{O_2C}.\ \overrightarrow{O_2d} \text{ will give velocity of point D.}$$

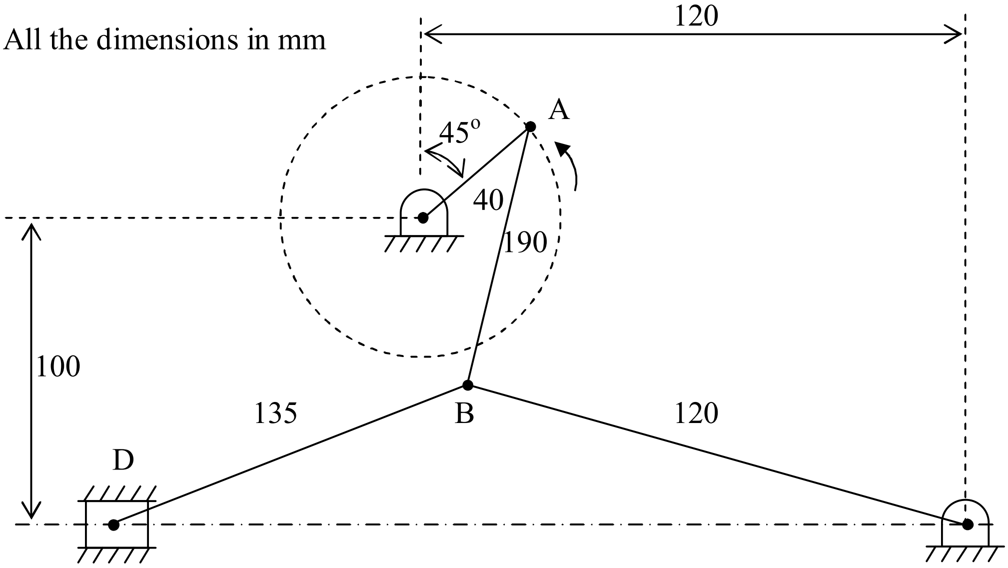
- From d draw a line $\perp^r$ to link DR and from O_1O_2. Draw a line along the line of stroke of Ram R (horizontal), These two lines will intersect at point r $\overrightarrow{O_2r}$ will give the velocity of Ram R.
- To determine the angular velocity of link O_2D determine $V_d = \overrightarrow{O_2d}$.

We know that $V_d = \omega_{O2D} \times O_2D$.

$$\therefore\ \omega_{O^2d} = \frac{\overrightarrow{O_2d}}{O_2D} \text{ r/s}$$

- **Problem 4:** Figure below shows a toggle mechanisms in which the crank OA rotates at 120 rpm. Find the velocity and acceleration of the slider D.

- **Solution:**



Configuration Diagram

Step 1: Draw the configuration diagram choosing a suitable scal.

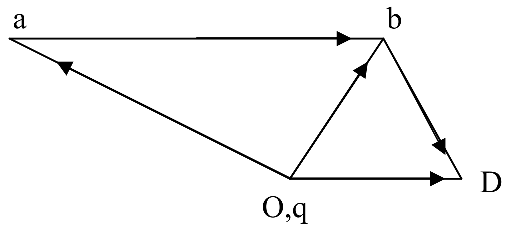
Step 2: Determine velocity of point A with respect to O.

$$V_{ao} = \omega_{OA} \text{ x } OA$$

$$V_{ao} = \frac{2\pi \text{ x } 120}{60} = 0.4 = 5.024 \text{ m/s}$$

Step 3: Draw the velocity vector diagram.

- Choose a suitable scale
- Mark zero velocity points O,q
- Draw vector $\overrightarrow{oa} \perp^{r}$ to link OA and magnitude = 5.024 m/s.



Velocity vector diagram

- From a draw a line $\perp^r$ to AB and from q draw a line $\perp^r$ to QB to intersect at b.

  $\overrightarrow{ab} = V_{ba}$ and $\overrightarrow{qb} = V_{bq}$.

- Draw a line $\perp^r$ to BD from b from q draw a line along the slide to intersect at d.

  $\overrightarrow{dq} = V_d$ (slider velocity)

- **Problem 5:** A whitworth quick return mechanism shown in figure has the following dimensions of the links.

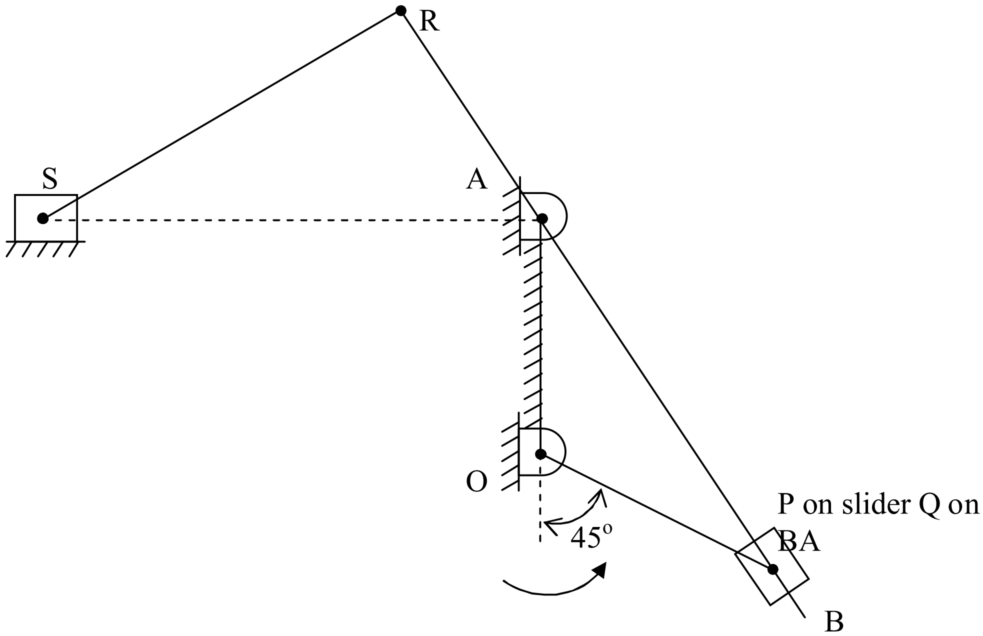
The crank rotates at an angular velocity of 2.5 r/s at the moment when crank makes an angle of 45^o with vertical. Calculate

a) the velocity of the Ram S
b) the velocity of slider P on the slotted level
c) the angular velocity of the link RS.

OP (crank) = 240 mm
OA = 150 mm
AR = 165 mm
RS = 430 mm

- **Solution:**

Step 1: To draw configuration diagram to a suitable scale.



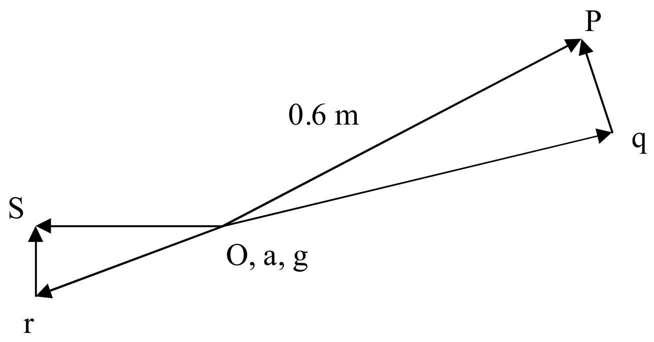
Configuration Diagram

Step 2: To determine the absolute velocity of point P.

$$V_P = \omega_{OP} \text{ x } OP$$

$$V_{ao} = \frac{2\pi \text{ x } 240}{60} \text{ x } 0.24 = 0.6 \, m/s$$

Step 3: Draw the velocity vector diagram by choosing a suitable scale.



Velocity vector diagram

- Draw $\overrightarrow{op}$ $\perp^r$ link OP = 0.6 m.

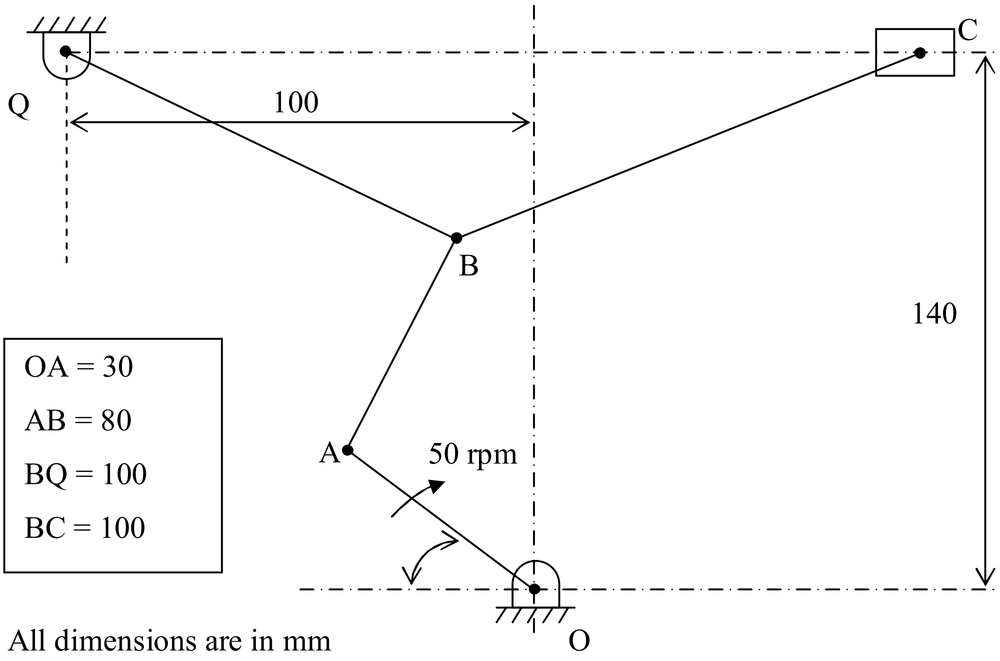
- From O, a, g draw a line $\perp^r$ to AP/AQ and from P draw a line along AP to intersect previously draw, line at q. $\overrightarrow{Pq}$ = Velocity of sliding.

  $\overrightarrow{aq}$ = Velocity of Q with respect to A.

  $V_{qa} = \overrightarrow{aq} =$

- Angular velocity of link RS = $\omega_{RS} = \frac{\overrightarrow{sr}}{SR}$ rad/sec

- **Problem 6:** A toggle mechanism is shown in figure along with the diagrams of the links in mm. find the velocities of the points B and C and the angular velocities of links AB, BQ and BC. The crank rotates at 50 rpm in the clockwise direction.



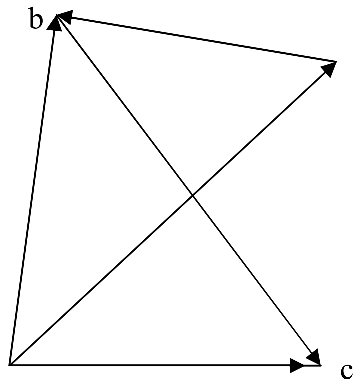
- **Solution**

<u>Step 1</u>: Draw the configuration diagram to a suitable scale.

<u>Step 2</u>: Calculate the magnitude of velocity of A with respect to O.

$$V_a = \omega_{OA} \text{ x } OA$$

$$V_a = \left(\frac{2\pi \text{ x } 50}{60}\right) \text{ x } 0.03 = 0.05\pi\, m/s = 0.1507\, m/s$$



Vector velocity diagram

Step 3: Draw the velocity vector diagram by choosing a suitable scale.

- Draw $\overrightarrow{Oa}$ $\perp^r$ to link OA = 0.15 m/s
- From a draw a link $\perp^r$ to AB and from O, q draw a link $\perp^r$ to BQ to intersect at b.

$$\overrightarrow{ab} = V_{ba} = \qquad \text{and } \overrightarrow{qb} = V_b = 0.13\,m/s$$

$$\omega_{ab} = \frac{\overrightarrow{ab}}{AB} = 0.74\,r/s\,(ccw) \quad \omega_{bq}\ \frac{\overrightarrow{qb}}{aB} = 1.3\ r/s\ (ccw)$$

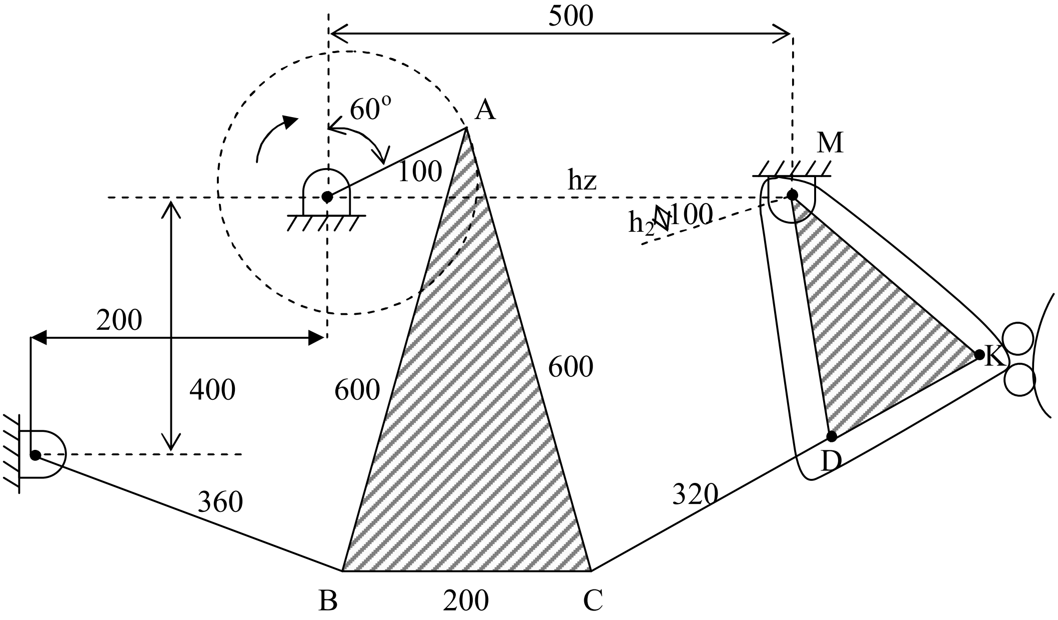
- From b draw a line $\perp^r$ to Be and from O, q these two lines intersect at C.

$$\overrightarrow{OC} = V_C = 0.106\,m/s$$

$$\overrightarrow{bC} = V_{Cb} =$$

$$\omega_{BC} = \frac{\overrightarrow{bc}}{BC} = 1.33\ r/s\ (ccw)$$

- **Problem 7:** The mechanism of a stone crusher has the dimensions as shown in figure in mm. If crank rotates at 120 rpm CW. Find the velocity of point K when crank OA is inclined at 30° to the horizontal. What will be the torque required at the crank to overcome a horizontal force of 40 kN at K.



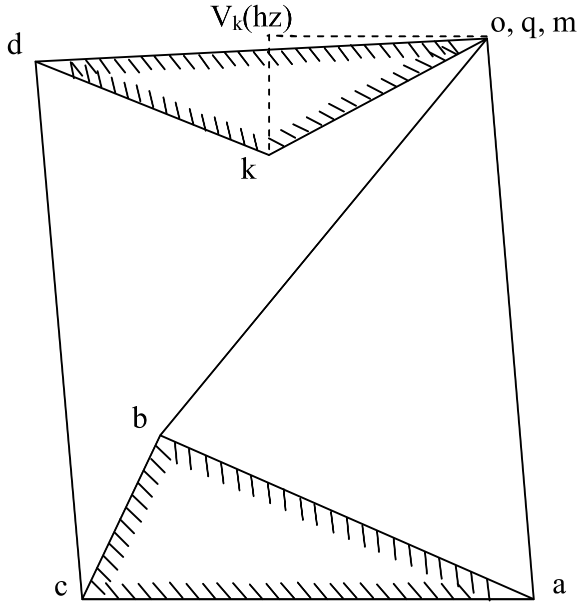
Configuration diagram

- **Solution**:

Step 1: Draw the configuration diagram to a suitable scale.

Step 2: Given speed of crank OA determine velocity of A with respect to 'o'.

$$V_a = \omega_{OA} \text{ x } OA = \left(\frac{2\pi \text{ x } 120}{60}\right) \text{ x } 0.1 = 1.26 \text{ m/s}$$



Velocity vector diagram

Step 3: Draw the velocity vector diagram by selecting a suitable scale.

- Draw $\overrightarrow{Oa}$ $\perp^r$ to link OA = 1.26 m/s
- From a draw a link $\perp^r$ to AB and from q draw a link $\perp^r$ to BQ to intersect at b.
- From b draw a line $\perp^r$ to BC and from a, draw a line $\perp^r$ to AC to intersect at c.
- From c draw a line $\perp^r$ to CD and from m draw a line $\perp^r$ to MD to intersect at d.
- From d draw a line $\perp^r$ to KD and from m draw a line $\perp^r$ to KM to x intersect the previously drawn line at k.
- Since we have to determine the torque required at OA to overcome a horizontal force of 40 kN at K. Draw a the horizontal line from o, q, m and c line $\perp^r$ to this line from k.

$$\therefore (\omega T)_{I/P} = (\omega T)_{O/P}$$

$$V = \omega_R \qquad T = F \text{ x } P \qquad F = \frac{T}{r}$$

$$\therefore \omega_{OA} T_{OA} = F_k V_k \text{ horizontal}$$

$$\therefore T_{OA} = \frac{F_k \; V_{k(hz)}}{\omega_{OA}}$$

$$T_{OA} = \frac{40000 \text{ X } 0.45}{12.6} = \quad \text{N-m}$$

- **Problem 8:** In the mechanism shown in figure link OA = 320 mm, AC = 680 mm and OQ = 650 mm.

Determine,

i) The angular velocity of the cylinder

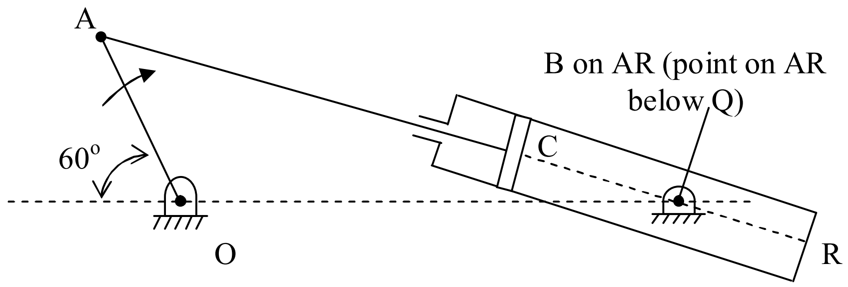
ii) The sliding velocity of the plunger

iii) The absolute velocity of the plunger

When the crank OA rotates at 20 rad/sec clockwise.

- **Solution**:

Step 1: Draw the configuration diagram.



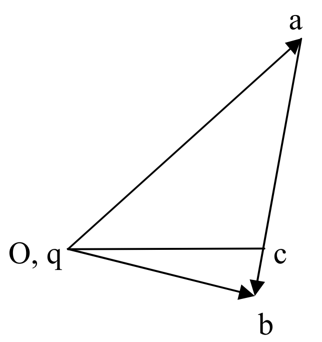
Step 2: Draw the velocity vector diagram

- Determine velocity of point A with respect to O.

  $V_a = \omega_{OA} \text{ x OA} = 20 \text{ x } 0.32 = 6.4$ m/s

- Select a suitable scale to draw the velocity vector diagram.

- Mark the zero velocity point. Draw vector $\overrightarrow{oa}$ $\perp^r$ to link OA equal to 6.4 m/s.



- From a draw a line $\perp^r$ to AB and from o, q, draw a line perpendicular to AB.
- To mark point c on $\overrightarrow{ab}$

$$\text{We know that } \frac{\overrightarrow{ab}}{\overrightarrow{ac}} = \frac{AB}{AC}$$

$$\therefore \overrightarrow{ac} = \frac{\overrightarrow{ab} \times AC}{AB} =$$

- Mark point c on $\overrightarrow{ab}$ and joint this to zero velocity point.
- Angular velocity of cylinder will be.

$$\omega_{ab} = \frac{V_{ab}}{AB} = 5.61 \text{ rad/sec (c}\omega)$$

- Studying velocity of player will be

$$\overrightarrow{qb} = 4.1 \text{ m/s}$$

- Absolute velocity of plunger = $\dfrac{\overrightarrow{OC}}{\overrightarrow{qc}}$ = 4.22 m/s

- **Problem 9:** In a swiveling joint mechanism shown in figure link AB is the driving crank which rotates at 300 rpm clockwise. The length of the various links are:

Determine,

i) The velocity of slider block S

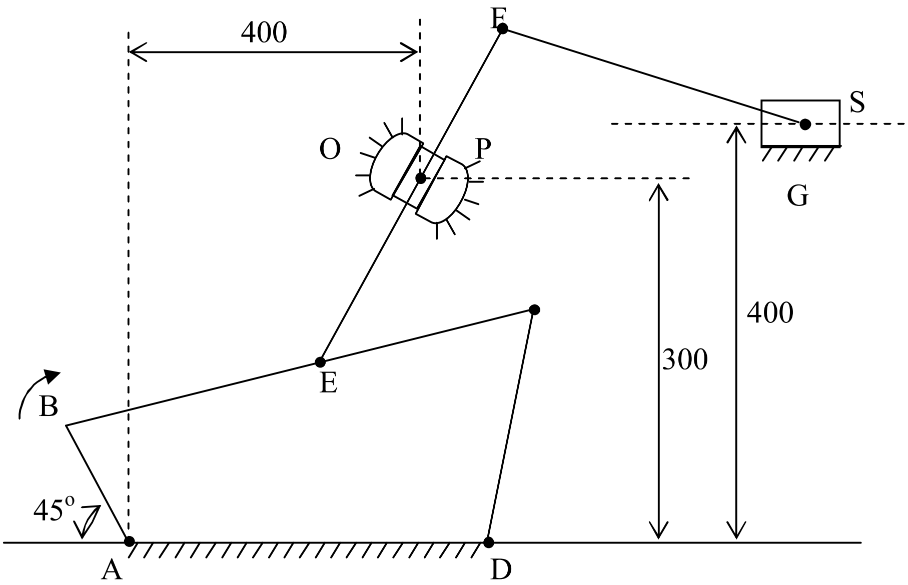
ii) The angular velocity of link EF

iii) The velocity of link EF in the swivel block.

AB = 650 mm
AB = 100 mm
BC = 800 mm
DC = 250 mm
BE = CF
EF = 400 mm
OF = 240 mm
FS = 400 mm

- **Solution:**

Step 1: Draw the configuration diagram.



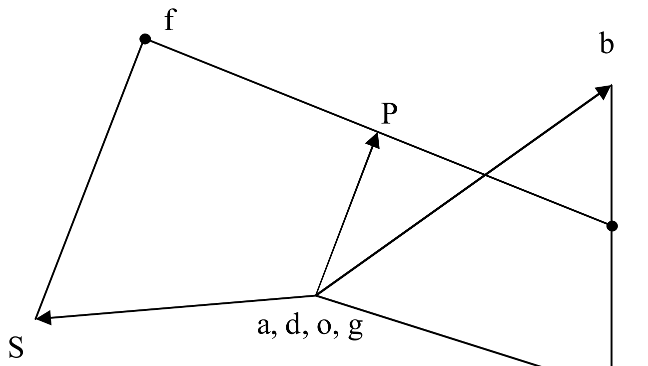
Step 2: Determine the velocity of point B with respect to A.

$V_b = \omega_{BA} \times BA$

$$V_b = \frac{2\pi \times 300}{60} \times 0.1 = 3.14 \text{ m/s}$$

Step 3: Draw the velocity vector diagram choosing a suitable scale.

- Mark zero velocity point a, d, o, g.



Velocity vector diagram

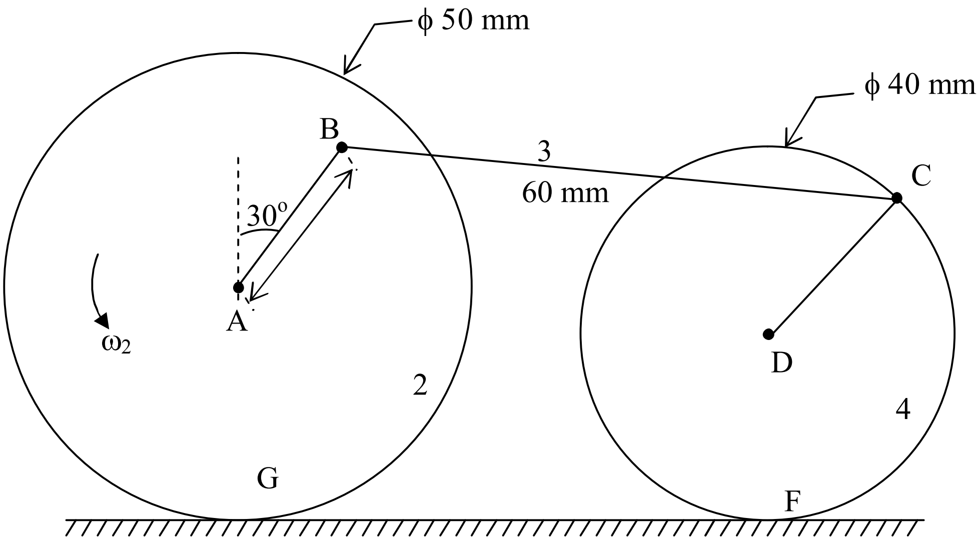
- From 'a' draw a line $\perp^r$ to AB and equal to 3.14 m/s.
- From 'b' draw a line $\perp^r$ to DC to intersect at C.

- Mark a point 'e' on vector bc such that

  $\overrightarrow{be} = \overrightarrow{bc} \times \frac{BE}{BC}$

- From 'e' draw a line $\perp^r$ to PE and from 'a,d' draw a line along PE to intersect at P.
- Extend the vector ep to ef such that $\overrightarrow{ef} = \frac{\overrightarrow{ef}}{EP} \times EF$
- From 'f' draw a line $\perp^r$ to Sf and from zero velocity point draw a line along the slider 'S' to intersect the previously drawn line at S.
- Velocity of slider $\overrightarrow{gS} = 2.6\,m/s$. Angular Velocity of link EF.
- Velocity of link F in the swivel block = $\overrightarrow{OP} = 1.85\ m/s$.

- **Problem 10:** Figure shows two wheels 2 and 4 which rolls on a fixed link 1. The angular uniform velocity of wheel is 2 is 10 rod/sec. Determine the angular velocity of links 3 and 4, and also the relative velocity of point D with respect to point E.



- **Solution:**

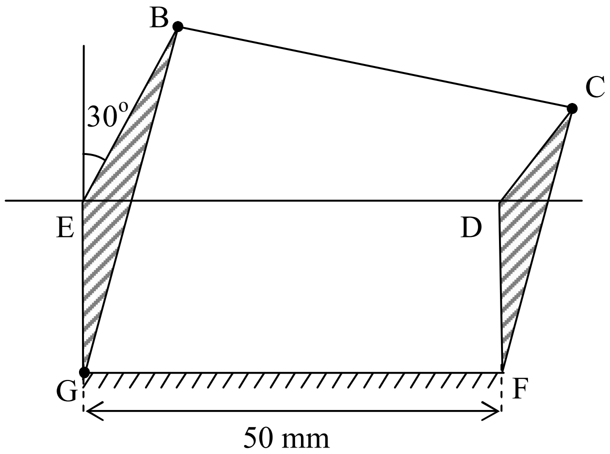
Step 1: Draw the configuration diagram.

Step 2: Given ω_2 = 10 rad/sec. Calculate velocity of B with respect to G.

$V_b = \omega_2 \times BG$

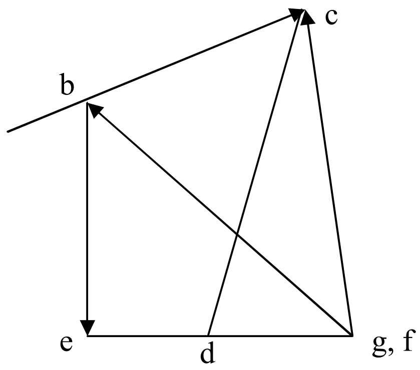
$V_b = 10 \times 43 = 430$ mm/sec.

Step 3: Draw the velocity vector diagram by choosing a suitable scale.



Redrawn configuration diagram

- **Velocity vector diagram**



  - Draw $\overrightarrow{gb} = 0.43$ m/s $\perp^r$ to BG.
  - From b draw a line $\perp^r$ to BC and from 'f' draw a line $\perp^r$ to CF to intersect at C.
  - From b draw a line $\perp^r$ to BE and from g, f draw a line $\perp^r$ to GE to intersect at e.
  - From c draw a line $\perp^r$ to CD and from f draw a line $\perp^r$ to FD to intersect at d.

- **Problem 11**: For the mechanism shown in figure link 2 rotates at constant angular velocity of 1 rad/sec construct the velocity polygon and determine.

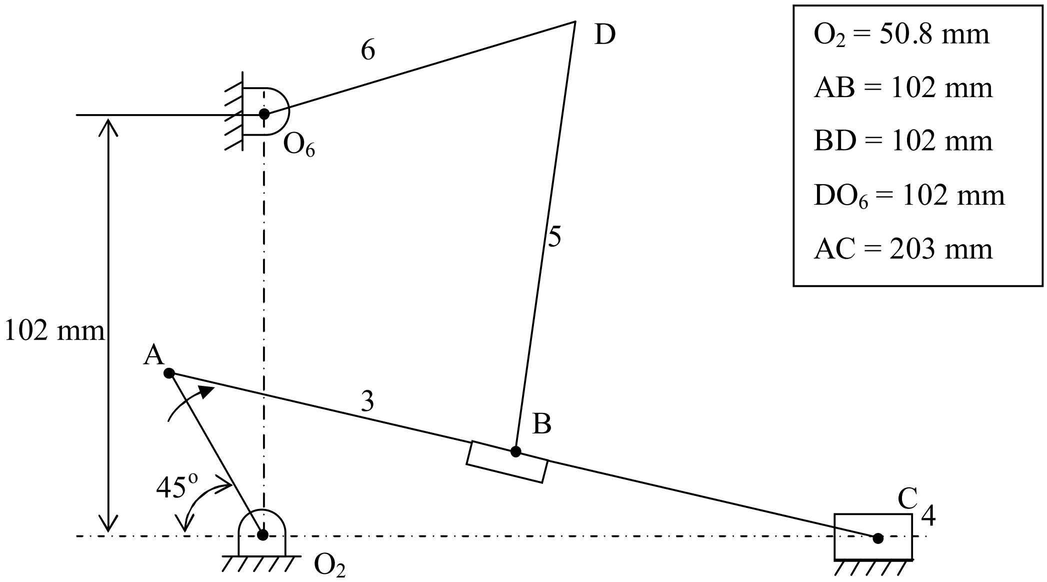
  i) Velocity of point D.

  ii) Angular velocity of link BD.

  iii) Velocity of slider C.

- **Solution**:

<u>Step 1</u>: Draw configuration diagram.

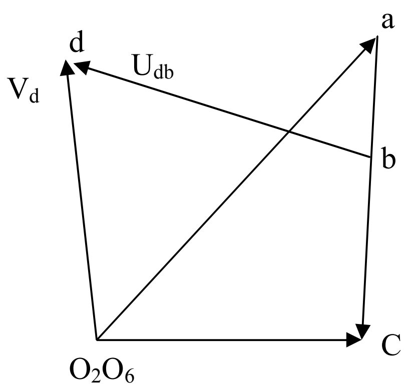


Step 2: Determine velocity of A with respect to O_2.

$V_b = \omega_2 \text{ x } O_2A$

$V_b = 1 \text{ x } 50.8 = 50.8$ mm/sec.

Step 3: Draw the velocity vector diagram, locate zero velocity points O_2O_6.



- From O_2, O_6 draw a line $\perp^r$ to O_2A in the direction of rotation equal to 50.8 mm/sec.
- From a draw a line $\perp^r$ to Ac and from O_2, O_6 draw a line along the line of stocks of c to intersect the previously drawn line at c.
- Mark point b on vector ac such that $\overrightarrow{ab} = \frac{\overrightarrow{ab}}{AC} \text{ x } AB$
- From b draw a line $\perp^r$ to BD and from O_2, O_6 draw a line $\perp^r$ to O_6D to intersect at d.

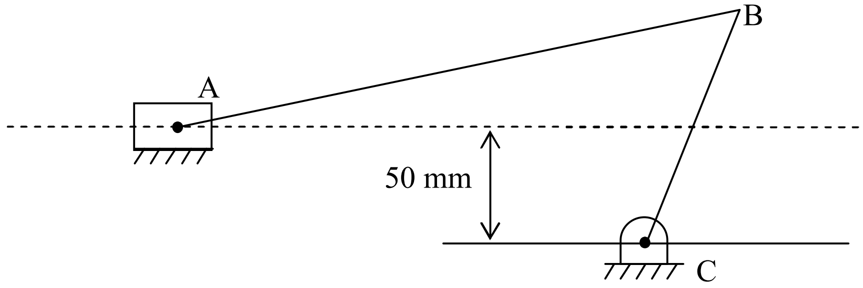
Step 4: $V_d = \overrightarrow{O_6 d} = 32$ mm/sec

$\omega_{bd} = \dfrac{\overrightarrow{bd}}{BD} =$

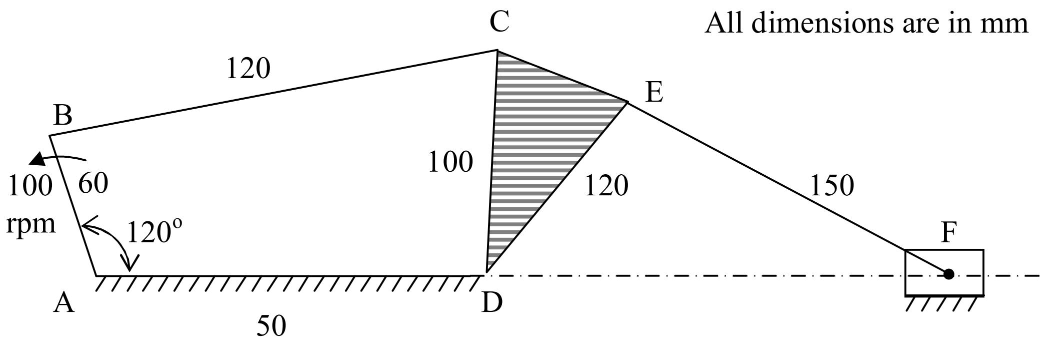
$V_c = \overrightarrow{O_2 C} =$

ADDITIONAL PROBLEMS FOR PRACTICE

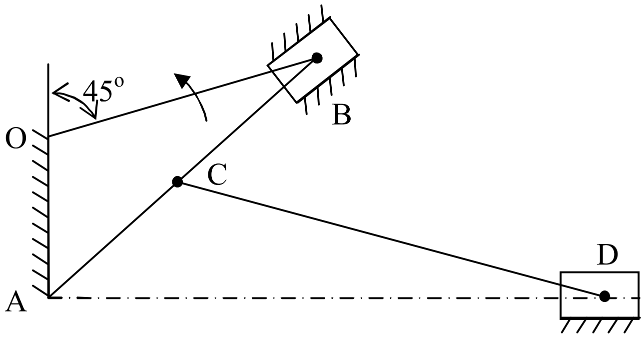
- **Problem 1**: In a slider crank mechanism shown in offset by a perpendicular distance of 50 mm from the centre C. AB and BC are 750 mm and 200 mm long respectively crank BC is rotating eω at a uniform speed of 200 rpm. Draw the velocity vector diagram and determine velocity of slider A and angular velocity of link AB.



- **Problem 2**: For the mechanism shown in figure determine the velocities at points C, E and F and the angular velocities of links, BC, CDE and EF.

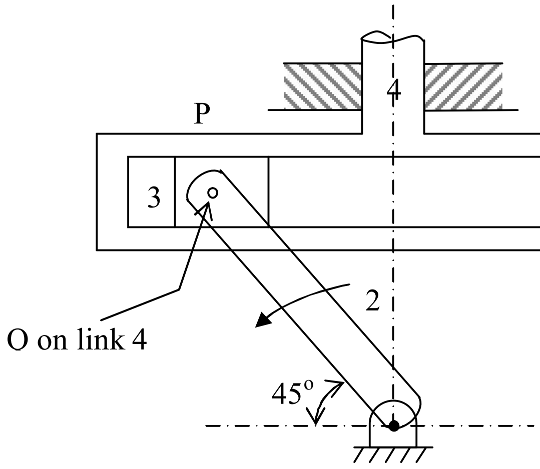


- The crank op of a crank and slotted lever mechanism shown in figure rotates at 100 rpm in the CCW direction. Various lengths of the links are OP = 90 mm, OA = 300 mm, AR = 480 mm and RS = 330 mm. The slider moves along an axis perpendicular to $\perp^r$ AO and in 120 mm from O. Determine the velocity of the slider when $\underline{|AOP}$ is 135° and also mention the maximum velocity of slider.

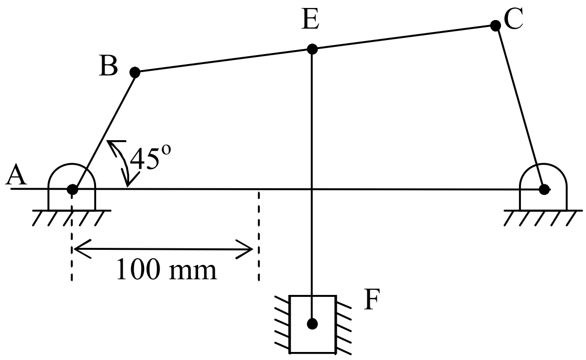


45°
O
B
C
D
A

- **Problem 4**: Find the velocity of link 4 of the scotch yoke mechanism shown in figure. The angular speed of link 2 is 200 rad/sec CCW, link O_2P = 40 mm.



- **Problem 5**: In the mechanism shown in figure link AB rotates uniformly in Cω direction at 240 rpm. Determine the linear velocity of B and angular velocity of EF.



AB = 160 mm
BC = 160 mm
CD = 100 mm
AD = 200 mm
EF = 200 mm
CE = 40 mm

II Method

- **Instantaneous Method**

To explain instantaneous centre let us consider a plane body P having a non-linear motion relative to another body q consider two points A and B on body P having velocities as V_a and V_b respectively in the direction shown.

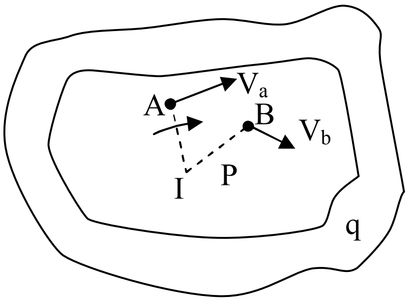


Fig. 1

If a line is drawn $\perp^r$ to V_a, at A the body can be imagined to rotate about some point on the line. Thirdly, centre of rotation of the body also lies on a line $\perp^r$ to the direction of V_b at B. If the intersection of the two lines is at I, the body P will be rotating about I at that instant. The point I is known as the instantaneous centre of rotation for the body P. The position of instantaneous centre changes with the motion of the body.

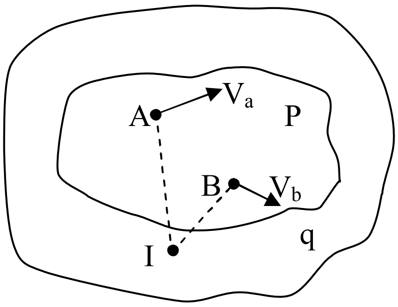


Fig. 2

In case of the $\perp^r$ lines drawn from A and B meet outside the body P as shown in Fig 2.

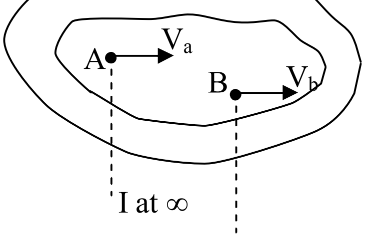


Fig. 3

If the direction of V_a and V_b are parallel to the $\perp^r$ at A and B met at ∞. This is the case when the body has linear motion.

- **Number of Instantaneous Centers**

The number of instantaneous centers in a mechanism depends upon number of links. If N is the number of instantaneous centers and n is the number of links.

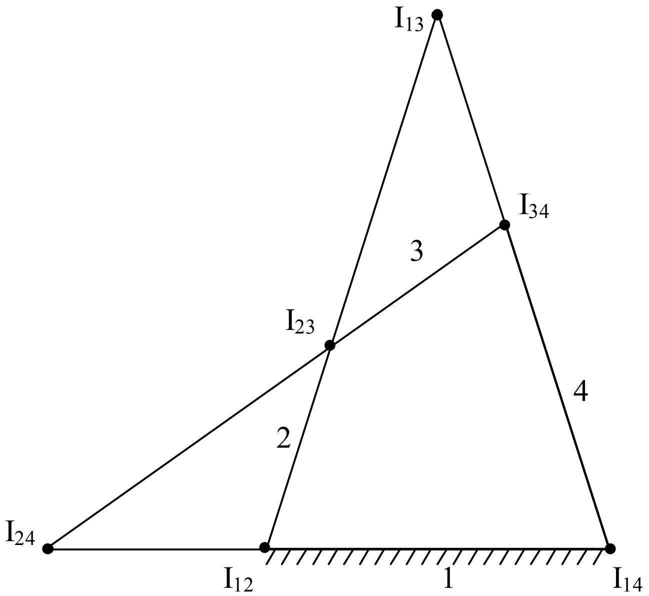
$$N = \frac{n(n-1)}{2}$$

- **Types of Instantaneous Centers**

There are three types of instantaneous centers namely fixed, permanent and neither fixed nor permanent.

Example: Four bar mechanism. n = 4.

$$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$



Fixed instantaneous center I_{12}, I_{14}

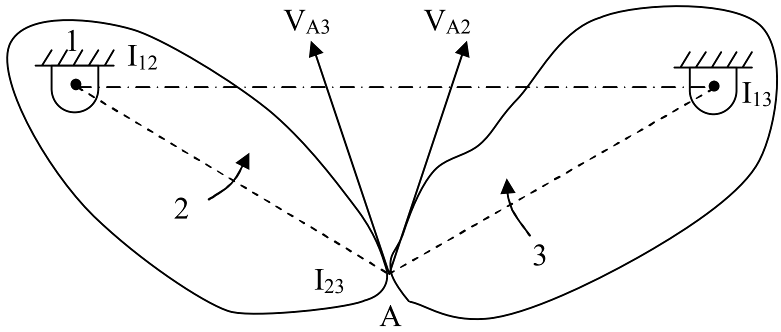
Permanent instantaneous center I_{23}, I_{34}

Neither fixed nor permanent instantaneous center I_{13}, I_{24}

- **Arnold Kennedy theorem of three centers:**

Statement: If three bodies have motion relative to each other, their instantaneous centers should lie in a straight line.

Proof:



Consider a three link mechanism with link 1 being fixed link 2 rotating about I_{12} and link 3 rotating about I_{13}. Hence, I_{12} and I_{13} are the instantaneous centers for link 2 and link 3. Let us assume that instantaneous center of link 2 and 3 be at point A i.e. I_{23}. Point A is a coincident point on link 2 and link 3.

Considering A on link 2, velocity of A with respect to I_{12} will be a vector V_{A2} $\perp^r$ to link A I_{12}. Similarly for point A on link 3, velocity of A with respect to I_{13} will be $\perp^r$ to A I_{13}. It is seen that velocity vector of V_{A2} and V_{A3} are in different directions which is impossible. Hence, the instantaneous center of the two links cannot be at the assumed position.

It can be seen that when I_{23} lies on the line joining I_{12} and I_{13} the V_{A2} and V_{A3} will be same in magnitude and direction. Hence, for the three links to be in relative motion all the three centers should lie in a same straight line. Hence, the proof.

Steps to locate instantaneous centers:

Step 1: Draw the configuration diagram.

Step 2: Identify the number of instantaneous centers by using the relation $N = \frac{(n-1)n}{2}$.

Step 3: Identify the instantaneous centers by circle diagram.

Step 4: Locate all the instantaneous centers by making use of Kennedy's theorem.

To illustrate the procedure let us consider an example.

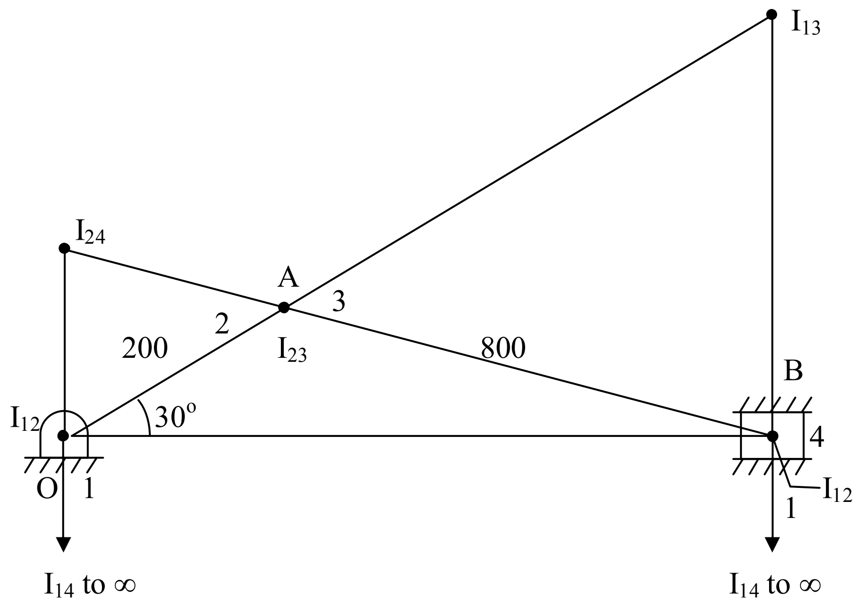
A slider crank mechanism has lengths of crank and connecting rod equal to 200 mm and 200 mm respectively locate all the instantaneous centers of the mechanism for the position of the crank when it has turned through 30° from IOC. Also find velocity of slider and angular velocity of connecting rod if crank rotates at 40 rad/sec.

Step 1: Draw configuration diagram to a suitable scale.

Step 2: Determine the number of links in the mechanism and find number of instantaneous centers.

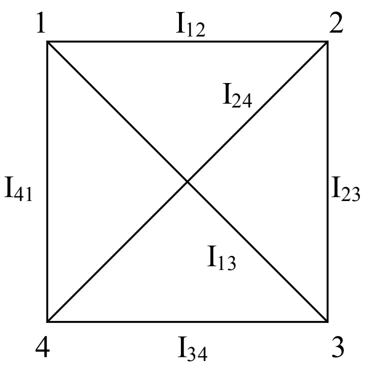
$$N = \frac{(n-1)n}{2}$$

$$n = 4 \text{ links} \qquad N = \frac{4(4-1)}{2} = 6$$

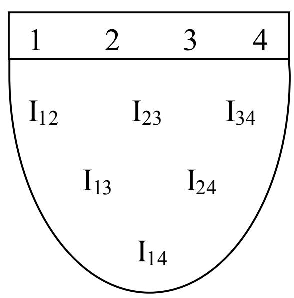


Step 3: Identify instantaneous centers.

- Suit it is a 4-bar link the resulting figure will be a square.



OR



- Locate fixed and permanent instantaneous centers. To locate neither fixed nor permanent instantaneous centers use Kennedy's three centers theorem.

Step 4: Velocity of different points.

$$V_a = \omega_2 \, AI_{12} = 40 \text{ x } 0.2 = 8 \text{ m/s}$$

also $V_a = \omega_2 \text{ x } A_{13}$

$$\therefore \omega_3 = \frac{V_a}{AI_{13}}$$

$V_b = \omega_3 \text{ x } BI_{13}$ = Velocity of slider.

- **Problem 2**:

A four bar mechanisms has links AB = 300 mm, BC = CD = 360 mm and AD = 600 mm. Angle $\angle BAD = 60^\circ$. Crank AB rotates in Cω direction at a speed of 100 rpm. Locate all the instantaneous centers and determine the angular velocity of link BC.

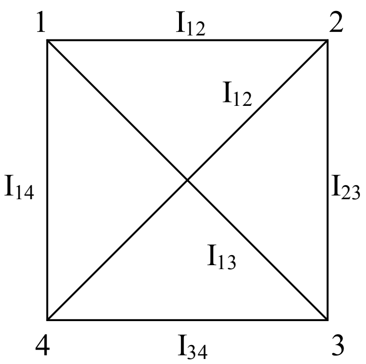
- **Solution:**

Step 1: Draw the configuration diagram to a suitable scale.

Step 2: Find the number of Instantaneous centers

$$N = \frac{(n-1)n}{2} = \frac{4(4-1)}{2} = 6$$

Step 3: Identify the IC's by circular method or book keeping method.



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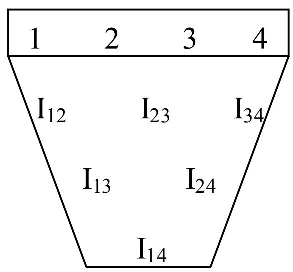
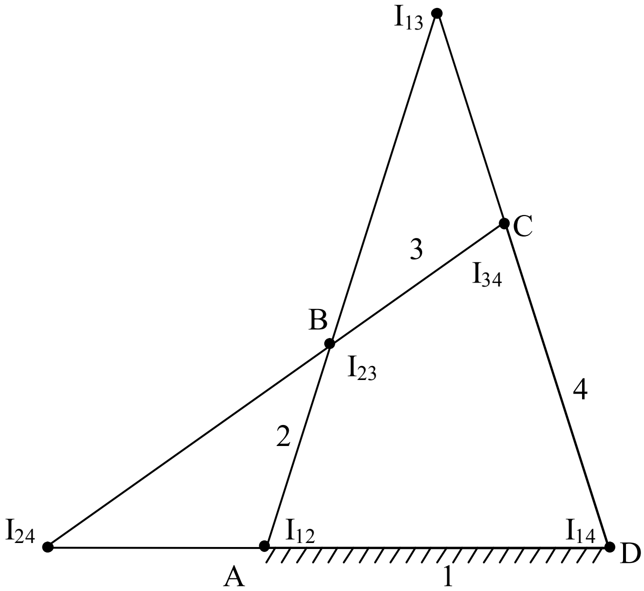


Step 4: Locate all the visible IC's and locate other IC's by Kennedy's theorem.



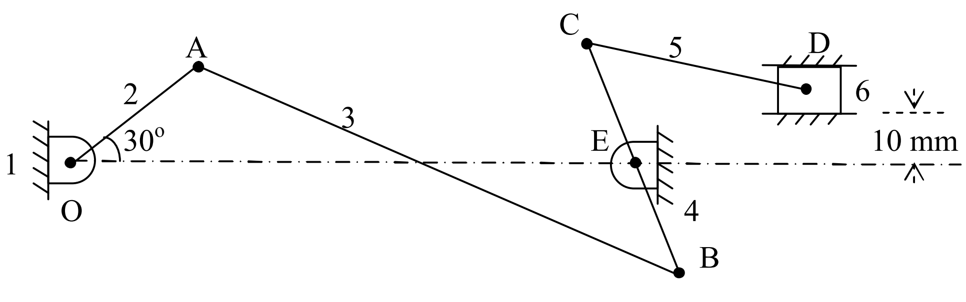
$$V_b = \omega_2 \text{ x } BI_{12} = \frac{2\pi \text{ x } 100}{60} \text{ x } 0.3 = \text{m/sec}$$

Also $V_b = \omega_3 \text{ x } BI_{13}$

$$\omega_3 = \frac{V_b}{BI_{13}} = \text{rad/sec}$$

- For a mechanism in figure crank OA rotates at 100 rpm clockwise using I.C. method determine the linear velocities of points B, C, D and angular velocities of links AB, BC and CD.

OA = 20 cm	AB = 150 cm	BC = 60 cm
CD = 50 cm	BE = 40 cm	OE = 135 cm



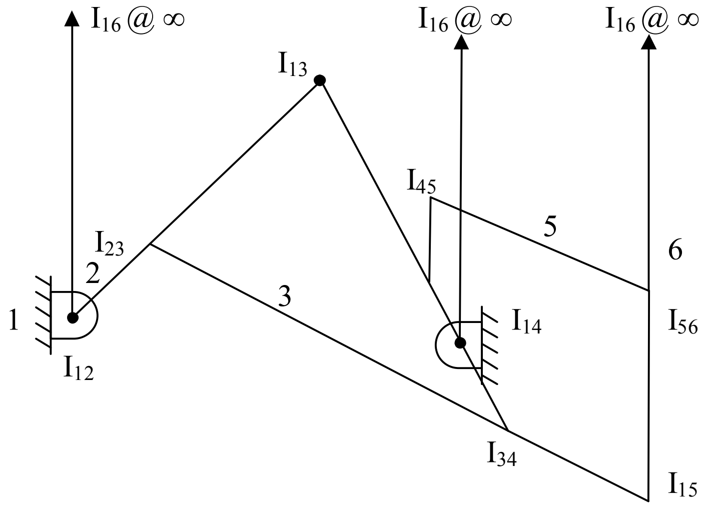
$V_a = \omega_{OA} \text{ x } OA$

$$V_a = \frac{2\pi \text{ x } 100}{60} \text{ x } 0.2 = 2.1 \text{ m/s}$$

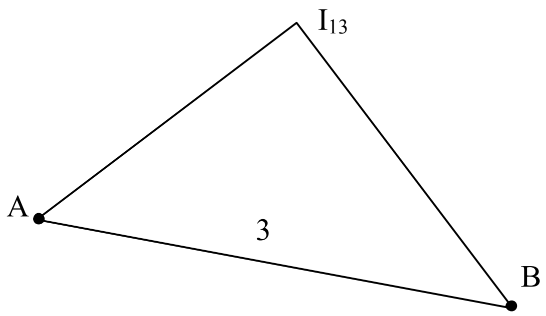
n = 6 links

$$N = \frac{n(n-1)}{2} = 15$$

1	2	3	4	5	6	5
12	23	34	45	56		4
13	24	35	46			3
14	25	36				2
15	26					1
16						---
						15



Link 3

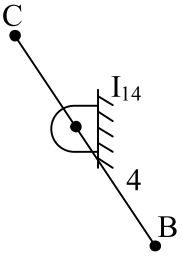


$$V_a = \omega_3 AI_{13}$$

$$\omega_3 = \frac{V_a}{AI_{13}} = 2.5\ \text{rad/sec}$$

$V_b = \omega_3 \times BI_{13} = 2.675$ m/s

Link 4

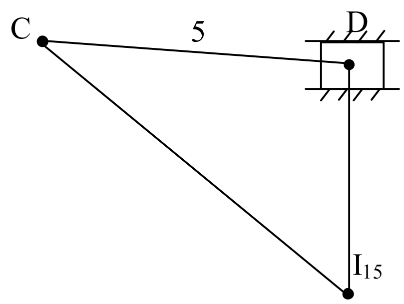


Also $V_b = \omega_4 \times BI_{14}$

$$\omega_4 = \frac{V_b}{BI_{14}} = 6.37 \text{ rad/sec}$$

$V_C = \omega_4 \times CI_{14} = 1.273$ m/s

Link 5



$V_C = \omega_5 \times CI_{15}$

$$\omega_5 = \frac{V_C}{AI_{15}} = 1.72 \text{ rad/sec}$$

$V_d = \omega_5 \times DI_{15} = 0.826$ m/s

Answers
$V_b = 2.675$ m/s
$V_C = 1.273$ m/s
$V_d = 0.826$ m/s
$\omega_{ab} = 2.5$ rad/sec
$\omega_{bc} = 6.37$ rad/sec
$\omega_{cd} = 1.72$ rad/sec

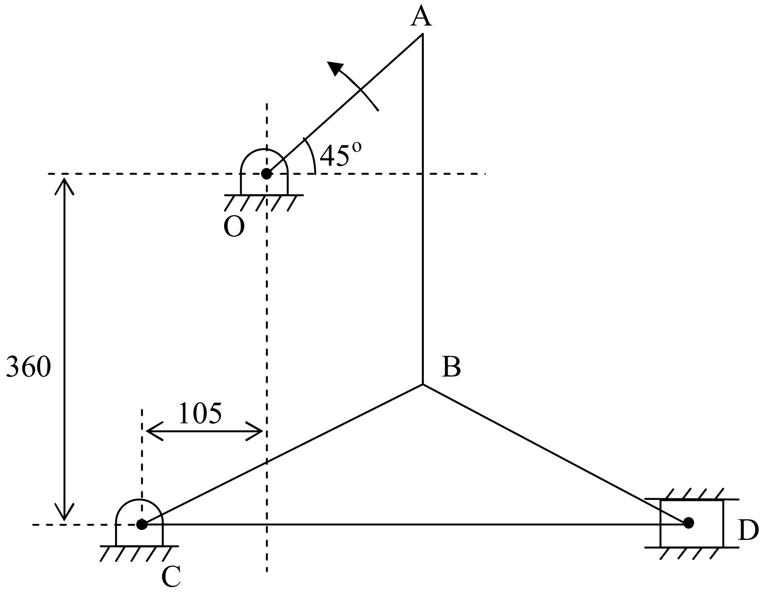
- In the toggle mechanism shown in figure the slider D is constrained to move in a horizontal path the crank OA is rotating in CCW direction at a speed of 180 rpm the dimensions of various links are as follows:

OA = 180 mm	CB = 240 mm
AB = 360 mm	BD = 540 mm

Find,

i) Velocity of slider

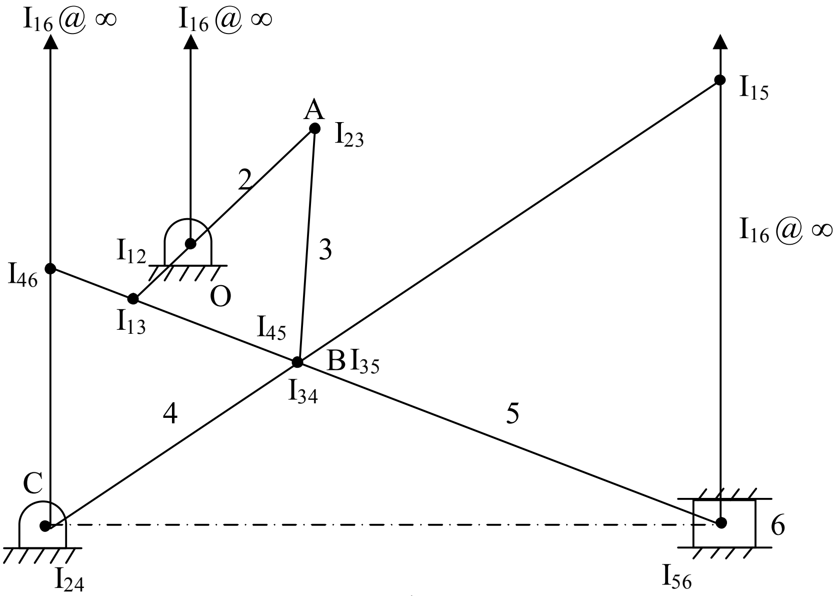
ii) Angular velocity of links AB, CB and BD.



n = 6 links

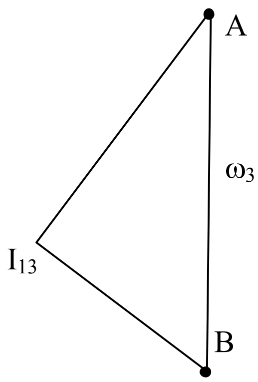
$$N = \frac{n(n-1)}{2} = 15$$

1	2	3	4	5	6	5
12	23	34	45	56		4
13	24	35	46			3
14	25	36				2
15	26					1
16						---
						15



$V_a = \omega_2 \times AI_{12} = 3.4 \text{ m/s}$

Link 3

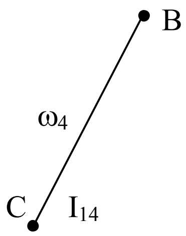


$V_a = \omega_3 \times AI_{13}$

$$\omega_3 = \frac{V_a}{AI_{13}} = 2.44 \text{ rad/sec}$$

$V_b = \omega_3 \times BI_{13}$

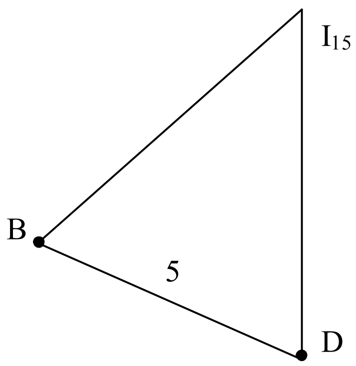
Link 4



$V_b = \omega_4 \times BI_{14}$

$$\omega_4 = \frac{V_b}{AI_{14}} = 11.875 \text{ rad/sec}$$

Link 5



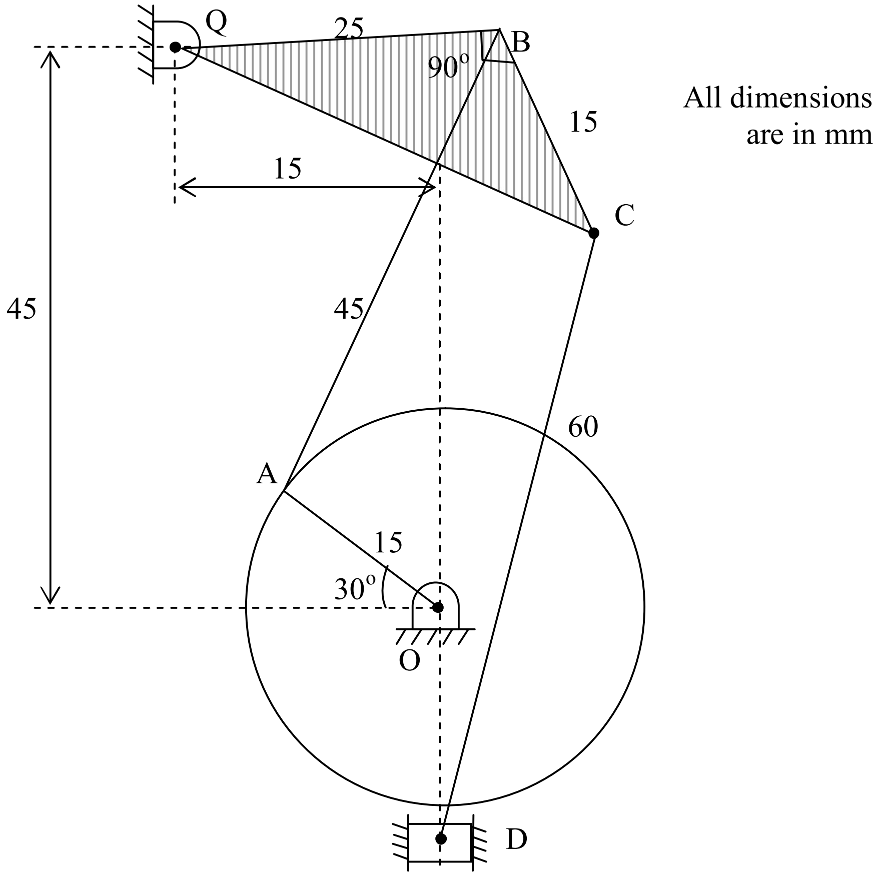
$V_b = \omega_5 \times BI_{15}$

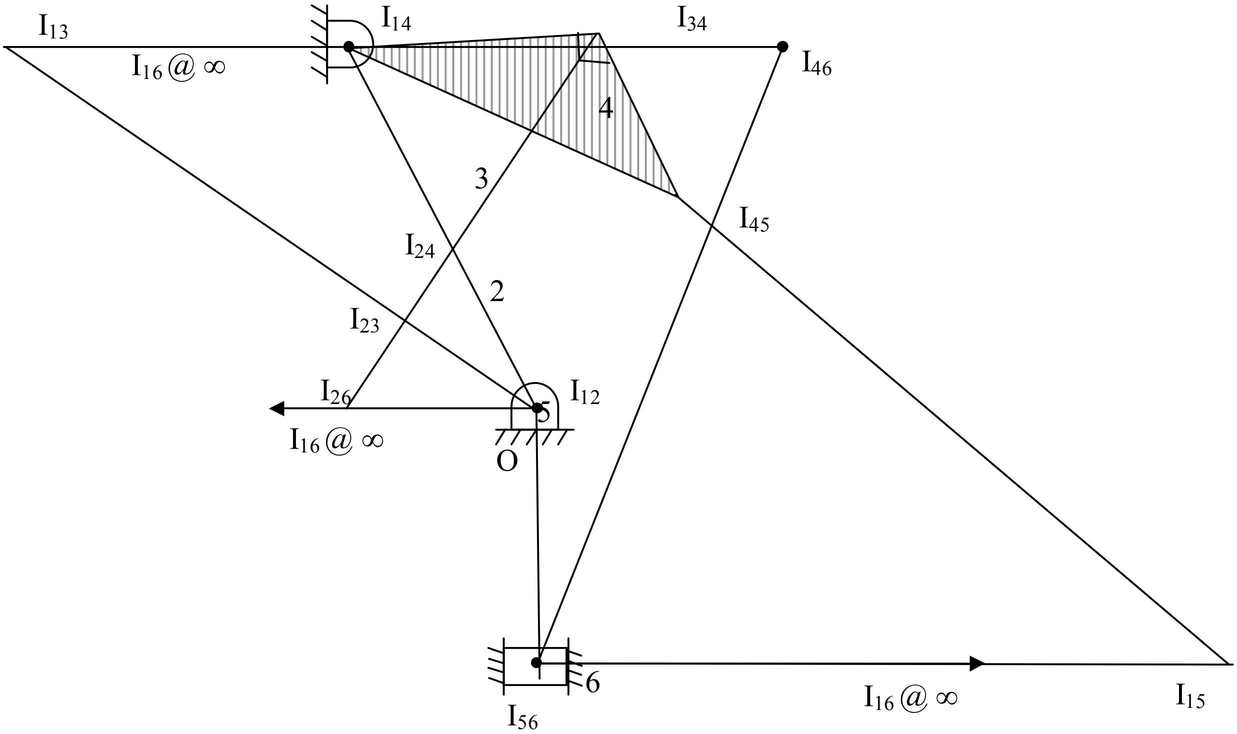
$$\omega_5 = \frac{V_b}{AI_{15}} = 4.37 \text{ rad/sec}$$

$V_d = \omega_5 \times DI_{15} = 2 \text{ m/s}$

Answers
$V_d = 2$ m/s
$\omega_{ab} = 2.44$ rad/sec
$\omega_{bc} = 11.875$ rad/sec
$\omega_{cd} = 4.37$ rad/sec

- Figure shows a six link mechanism. What will be tool D and the angular velocities of links BC and CD if crank rotates at 10 rad/sec.

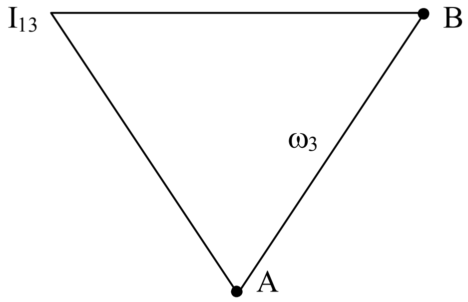

Q
25
B
90°
15
All dimensions
are in mm
15
C
45
45
60
A
15
30°
O
D



$V_a = \omega_2 \times AI_{12} = 10 \times 0.015$

$V_a = \omega_2 \times AI_{12} = 0.15$ m/s

Link 3

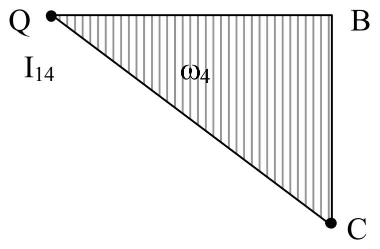


$V_a = \omega_3 \times AI_{13}$

$\omega_3 = \dfrac{V_a}{AI_{13}}$

$V_b = \omega_3 \times BI_{13}$

Link 4

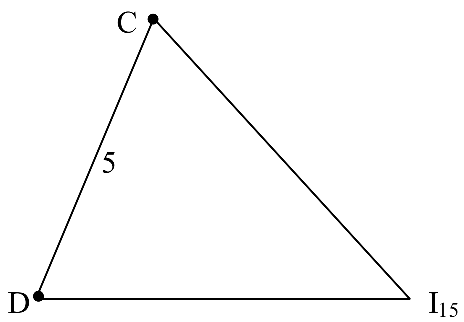


$V_b = \omega_4 \text{ x } BI_{14}$

$$\omega_4 = \frac{V_b}{BI_{14}} = 4.25 \text{ rad/sec}$$

$V_C = \omega_4 \text{ x } CI_{14}$

Link 5



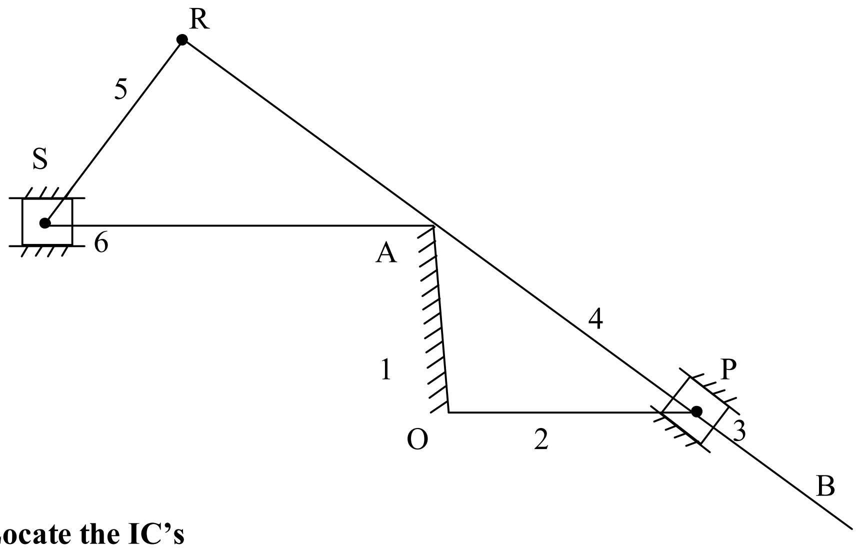
$V_C = \omega_5 \text{ x } CI_{15}$

$$\omega_5 = \frac{V_C}{AI_{15}} = 1.98 \text{ rad/sec}$$

$V_d = \omega_5 \text{ x } DI_{15} = 1.66$ m/s

Answers
$V_d = 1.66$ m/s
$\omega_{bc} = 4.25$ rad/sec
$\omega_{cd} = 1.98$ rad/sec

- A whitworth quick return mechanism shown in figure has a fixed link OA and crank OP having length 200 mm and 350 mm respectively. Other lengths are AR = 200 mm and RS = 40 mm. Find the velocity of the rotation using IC method when crank makes an angle of 120^o with fixed link and rotates at 10 rad/sec.



Locate the IC's

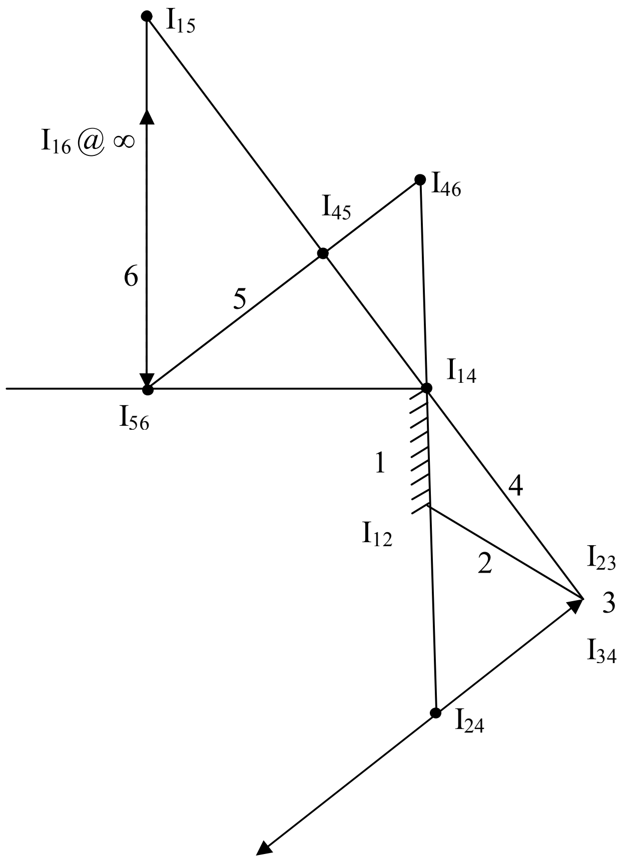
n = 6 links

$$N = \frac{n(n-1)}{2} = 15$$

1	2	3	4	5	6
12	23	34	45	56	
13	24	35	46		
14	25	36			
15	26				
16					

5
4
3
2
1

15



$V_P = \omega_2 \times OP = \ldots\ldots\ldots$ m/s

- **Acceleration Analysis**

Rate of change of velocity is acceleration. A change in velocity requires any one of the following conditions to be fulfilled:

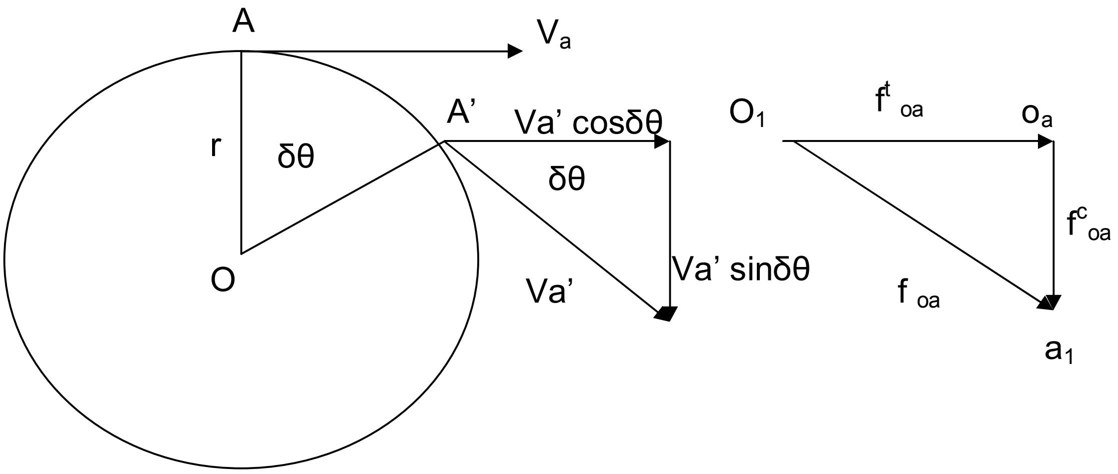
- Change in magnitude only
- Change in direction only
- Change in both magnitude and direction

When the velocity of a particle changes in magnitude and direction it has two component of acceleration.

1. Radial or centripetal acceleration

   $f^c = \omega^2 r$

   Acceleration is parallel to the link and acting towards centre.



Va' = (ω + α δ t) r
Velocity of A parallel to OA = 0
Velocity of A' parallel to OA = Va' sin δ θ
Therefore change in velocity = Va' sin δ θ – 0

Centripetal acceleration = $f^c = \dfrac{(\omega + \alpha\delta t) r \sin \delta\theta}{\delta t}$

as δt tends to Zero sin δ θ tends to δ θ

$$\therefore \frac{(\omega r \delta\theta + \alpha r \delta\theta\delta t)}{\delta t}$$

$f^c = \omega r\,(d\theta/dt) = \omega^2 r$

But V = ωr or ω = V/r

Hence, $\mathbf{f^c = \omega^2 r = V^2/r}$

2. Tnagential Acceleration:

Va' = (ω + α δ t) r
Velocity of A perpendicular to OA = Va
Velocity of A' perpendicular to OA = Va' cos δ θ
Therefore change in velocity = Va' cos δ θ – Va

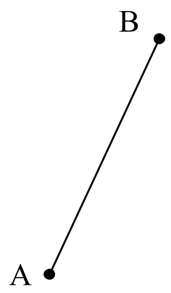
Tnagnetial acceleration = $f^t = \dfrac{(\omega + \alpha\delta t) r \cos \delta\theta - \omega r}{\delta t}$

as δt tends to Zero cos $\delta\theta$ tends to 1

$$\therefore \frac{(\omega r + \alpha r \delta t) - \omega r}{\delta t}$$

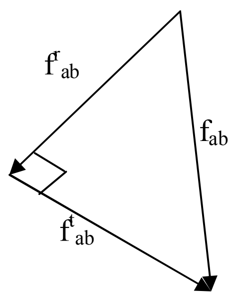
$$f^t = \alpha r$$

Example:



$f^C_{ab} = \omega^2 AB$

Acts parallel to BA and acts from B to A.

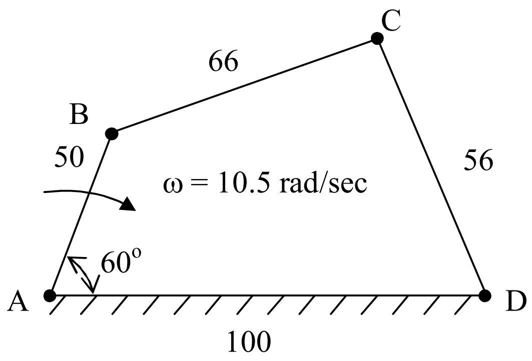


$f^t = \alpha BA$ acts $\perp^r$ to link.

$f_{BA} = f^r_{BA} + f^t_{BA}$

- **Problem 1**: Four bar mechanism. For a 4-bar mechanism shown in figure draw velocity and acceleration diagram.

All dimensions
are in mm



- **Solution**:

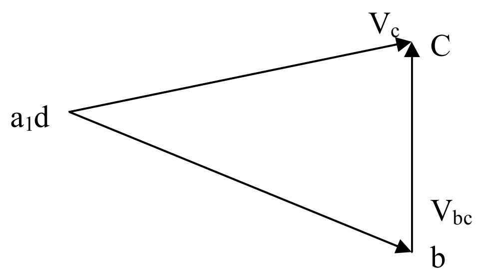
Step 1: Draw configuration diagram to a scale.

Step 2: Draw velocity vector diagram to a scale.

$V_b = \omega_2$ x AB

V_b = 10.5 x 0.05

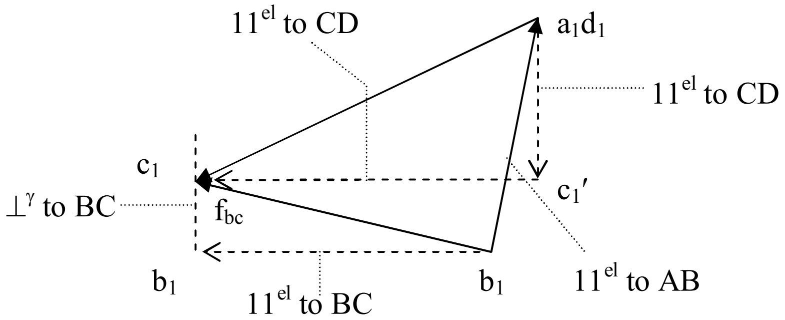
V_b = 0.525 m/s



Step 3: Prepare a table as shown below:

Sl. No.	Link	Magnitude	Direction	Sense
1.	AB	$f^c = \omega^2_{AB}r$ $f^c = (10.5)^2/0.525$ $f^c = 5.51$ m/s^2	Parallel to AB	→ A
2.	BC	$f^c = \omega^2_{BC}r$ $f^c = 1.75$ $f^t = \alpha r$	Parallel to BC $\perp^r$ to BC	→ B –
3.	CD	$f^c = \omega^2_{CD}r$ $f^c = 2.75$ $f^t =$?	Parallel to DC $\perp^r$ to DC	→ D –

Step 4: Draw the acceleration diagram.



- Choose a suitable scale to draw acceleration diagram.
- Mark the zero acceleration point a_1d_1.
- Link AB has only centripetal acceleration. Therefore, draw a line parallel to AB and toward A from a_1d_1 equal to 5.51 m/s^2 i.e. point b_1.
- From b_1 draw a vector parallel to BC points towards B equal to 1.75 m/s^2 (b^1_1).
- From b^1_1 draw a line $\perp^r$ to BC. The magnitude is not known.
- From a_1d_1 draw a vector parallel to AD and pointing towards D equal to 2.72 m/s^2 i.e. point c_1.
- From c^1_1 draw a line $\perp^r$ to CD to intersect the line drawn $\perp^r$ to BC at c_1, $\overrightarrow{d_1c_1} = f_{CD}$ and $\overrightarrow{b_1c_1} = f_{bc}$.

To determine angular acceleration.

$$\alpha_{BC} = \frac{f^t_{bc}}{BC} = \frac{\overrightarrow{c_1b^1_1}}{BC} = 34.09\,\text{rad/sec (CCW)}$$

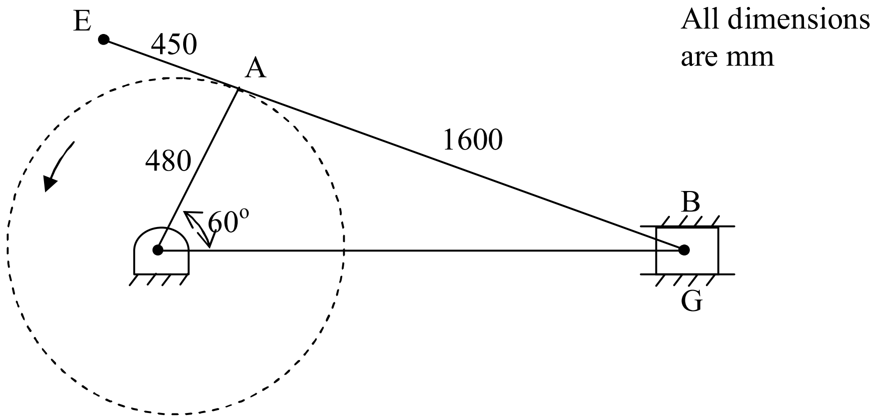
$$\alpha_{CD} = \frac{f^t_{cd}}{CD} = \frac{\overrightarrow{c_1c^1_1}}{CD} = 79.11\,\text{rad/sec (CCW)}$$

- **Problem 2**: For the configuration of slider crank mechanism shown in figure below.

Calculate

i) Acceleration of slider B.
ii) Acceleration of point E.
iii) Angular acceleration of link AB.
If crank OA rotates at 20 rad/sec CCW.

- **Solution**:



Step 1: Draw configuration diagram.

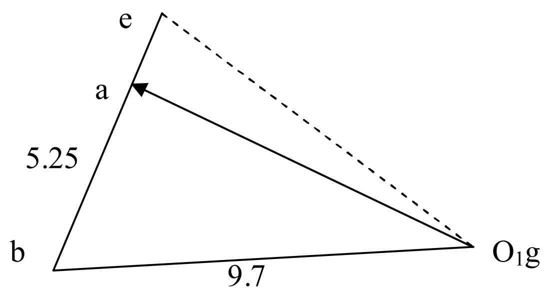
Step 2: Find velocity of A with respect to O.

$V_a = \omega_{OA} \times OA$

$V_a = 20 \times 0.48$

$V_a = 9.6$ m/s

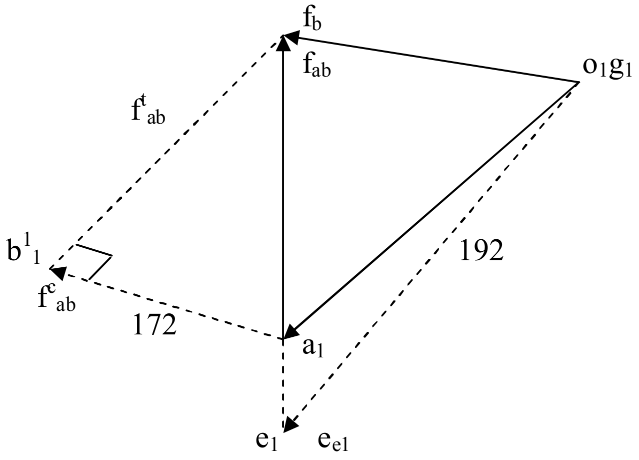
Step 4: Draw velocity vector diagram.



Step 4:

Sl. No.	Link	Magnitude	Direction	Sense
1.	OA	$f^c_{aO} = \omega^2_{OA} r = 192$	Parallel to OA	→ O
2.	AB	$f^c_{ab} = \omega^2_{ab} r = 17.2$ f^t_{ab} –	Parallel to AB $\perp^r$ to AB	→ A –
3.	Slider B	–	Parallel to Slider	–

Step 5: Draw the acceleration diagram choosing a suitable scale.



- Mark o_1g_1 (zero acceleration point)
- Draw $\overrightarrow{o_1g_1}$ = C acceleration of OA towards 'O'.
- From a_1 draw $a_1b^1{}_1 = 17.2\ m/s^2$ towards 'A' from $b^1{}_1$ draw a line $\perp^r$ to AB.
- From o_1g_1 draw a line along the slider B to intersect previously drawn line at b_1, $\overrightarrow{a_1b_1} = f_{ab}$

  $\overrightarrow{g_1b_1} = f_b = 72\ m/s^2$.
- Extend $\overrightarrow{a_1b_1} = \overrightarrow{a_1e_1}$ such that $\frac{\overrightarrow{a_1b_1}}{AB} = \frac{\overrightarrow{A_1R_1}}{AE}$.
- Join e_1 to δ_1g_1, $\overrightarrow{g_1e_1} = f_e = 236\ m/s^2$.
- $\alpha_{ab} = \frac{f^t_{ab}}{AB} = \frac{\overrightarrow{b_1b_1}}{AB} = \frac{167}{1.6} = 104\ rad/sec^2$ (CCW).

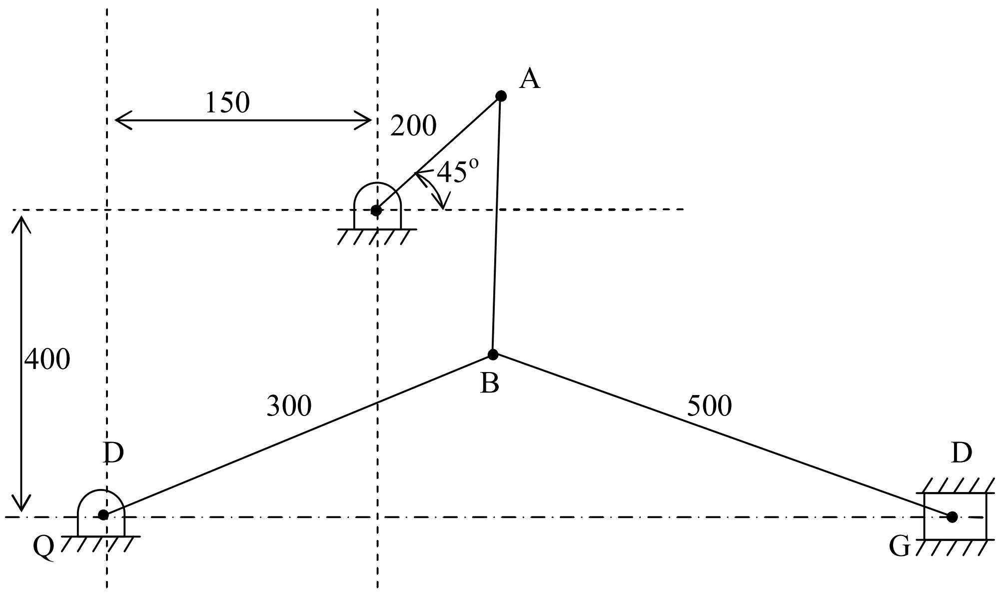
Answers:

$f_b = 72\ m/sec^2$

$f_e = 236\ m/sec^2$

$\alpha_{ab} = 104\ rad/sec^2$

- **Problem 3:** In a toggle mechanism shown in figure the crank OA rotates at 210 rpm CCW increasing at the rate of 60 rad/s^2.
  - Velocity of slider D and angular velocity of link BD.
  - Acceleration of slider D and angular acceleration of link BD.



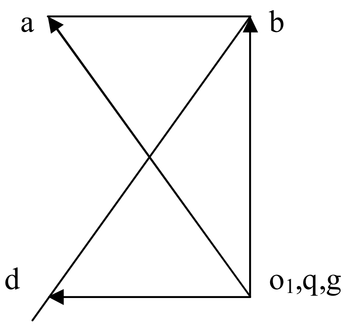
Step 1 Draw the configuration diagram to a scale.

Step 2 Find

$V_a = \omega_{OA} \text{ x } OA$

$$V_a = \frac{2\pi\left(210\right)}{60} \text{ x } 0.2 = 4.4 \text{ m/s}$$

Step 3: Draw the velocity vector diagram.



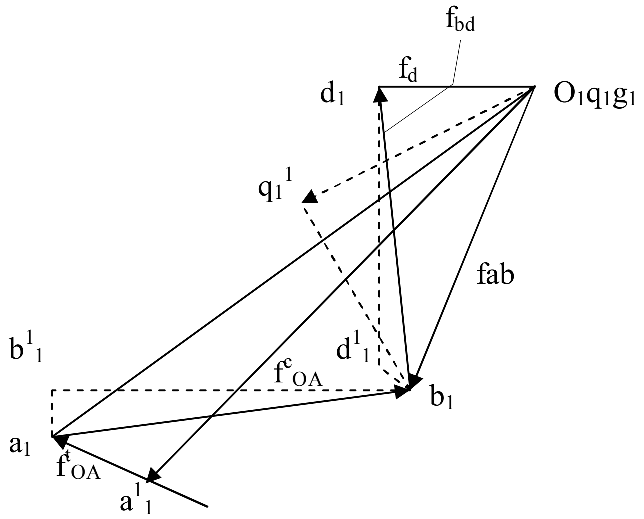
Step 4:

Sl. No.	Link	Magnitude m/s^2	Direction	Sense
1.	AO	$f^c_{aO} = \omega^2 r = 96.8$ $f^t_{aO} = \alpha r = 12$	Parallel to OA $\perp^r$ to OA	→ O –

2.	AB	$f^c_{ab} = \omega^2 r = 5.93$ $f^t_{ab} = \alpha r =$	Parallel to AB $\perp^r$ to AB	→ A –
3.	BQ	$f^c_{bq} = \omega^2 r = 38.3$ $f^t_{bq} = \alpha r =$	Parallel to BQ $\perp^r$ to BQ	→ Q –
4.	BD	$f^c_{bd} = \omega^2 r = 20$	$\perp^r$ to BD	→ B
5.	Slider D	$f^t_{bd} = \alpha r =$ –	$\perp^r$ to BD Parallel to slider motion	– –

Step 5: Draw the acceleration diagram choosing a suitable scale.

- Mark zero acceleration point.



- Draw $o_1a^1{}_1 = f^c_{OA}$ and $a^1{}_1a = f^t_{OA} \perp^r$ to OA from
- $\overrightarrow{o_1a_1} = f_a$
- From a_1 draw $\overrightarrow{a_1b_1} = f^c{}_{ab}$, from $b^1{}_1$ draw a line $\perp^r$ to AB.
- From $o_1q_1g_1$ draw $\overrightarrow{o_1q^1{}_1} = f^c_{bq}$ and from $q^1{}_1$ draw a line a line $\perp^r$ to BQ to intersect the previously drawn line at b_1

  $\overrightarrow{q_1b_1} = f_{bq}$ $\overrightarrow{a_1b_1} = f_{ab}$

- From b_1 draw a line parallel to BD = f^c_{bd} such that $\overrightarrow{b_1d^1{}_1} = f^c_{bd}$.
- From $d^1{}_1$ draw a line $\perp^r$ to BD, from $o_1q_1g_1$ draw a line along slider D to meet the previously drawn line at .

- $\overrightarrow{g_1d_1} = f_d = 16.4$ m/sec^2.
- $\overrightarrow{b_1d_1} = f_{bd} = 5.46$ m/sec^2.
- $\alpha_{BD} = \dfrac{f_{bd}}{BD} = \dfrac{5.46}{0.5}\ 109.2\ \text{rad}/\text{sec}^2$

Answers:

$V_d = 2.54$ m/s

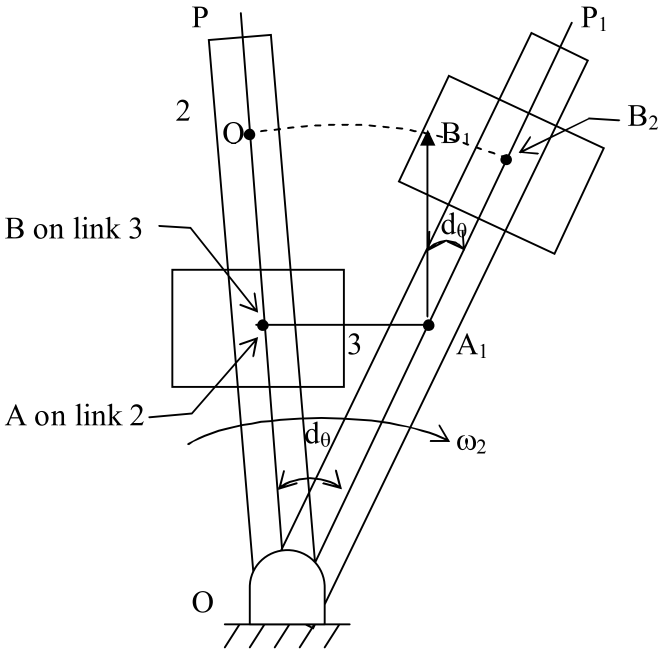
$\omega_{bd} = 6.32$ rad/s

$F_d = 16.4$ m/s^2

$\alpha_{bd} = 109.2$ rad/s^2

- **Coriolis Acceleration:** It has been seen that the acceleration of a body may have two components.
  - Centripetal acceleration and
  - Tangential acceleration.

However, in same cases there will be a third component called as corilis acceleration to illustrate this let us take an example of crank and slotted lever mechanisms.



Assume link 2 having constant angular velocity ω_2, in its motions from OP to OP_1 in a small interval of time δ_t. During this time slider 3 moves outwards from position B to B_2. Assume this motion also to have constant velocity $V_{B/A}$. Consider the motion of slider from B to B_2 in 3 stages.

1. B to A_1 due to rotation of link 2.
2. A_1 to B_1 due to outward velocity of slider $V_{B/A}$.
3. B_1 to B_2 due to acceleration $\perp^r$ to link 2 this component in the coriolis component of acceleration.

We have Arc B_1B_2 = Arc QB_2 – Arc QB_1

= Arc QB_2 – Arc AA_1

$\therefore$ Arc B_1B_2 = OQ dθ - AO dθ

= A_1B_1 dθ

$$= \mathbf{V_{B/A}\,\omega_2 dt^2}$$

The tangential component of velocity is $\perp^r$ to the link and is given by $V^t = \omega r$. In this case ω has been assumed constant and the slider is moving on the link with constant velocity. Therefore, tangential velocity of any point B on the slider 3 will result in uniform increase in tangential velocity. The equation $V^t = \omega r$ remain same but r increases uniformly i.e. there is a constant acceleration $\perp^r$ to rod.

$\therefore$ Displacement $B_1B_2 = \frac{1}{2}\,at^2$

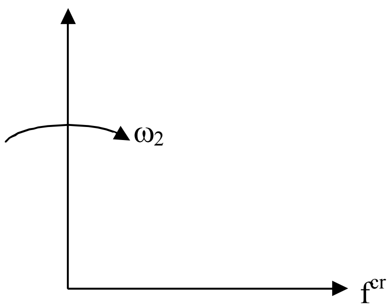
$= \frac{1}{2}\,f\,(dt)^2$

$\therefore\ \frac{1}{2}\,f\,(dt)^2 = V_{B/A}\,\omega_2\,dt^2$

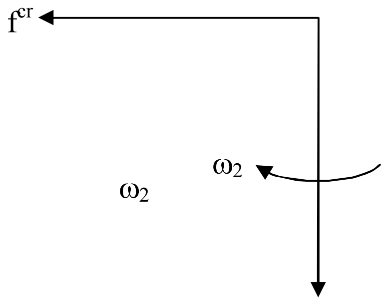
$\mathbf{f^{cr}_{B/A} = 2\omega_2\,V_{B/A}}$ **coriolis acceleration**

The direction of coriolis component is the direction of relative velocity vector for the two coincident points rotated at 90° in the direction of angular velocity of rotation of the link.

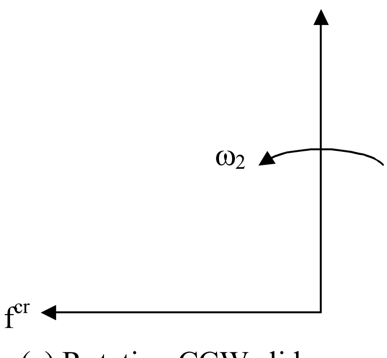
Figure below shows the direction of coriolis acceleration in different situation.



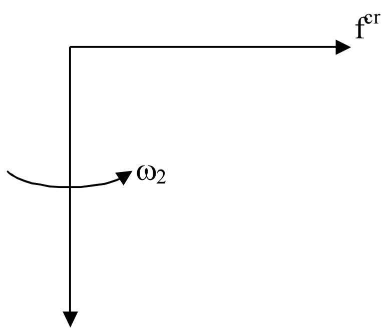
(a) Rotation CW slider moving up



(b) Rotation CW slider moving down



(c) Rotation CCW slider moving up



(d) Rotation CCW slider moving down

A quick return mechanism of crank and slotted lever type shaping machine is shown in Fig. the dimensions of various links are as follows.

O_1O_2 = 800 mm, O_1B = 300 mm, O_2D = 1300 mm and DR = 400 mm

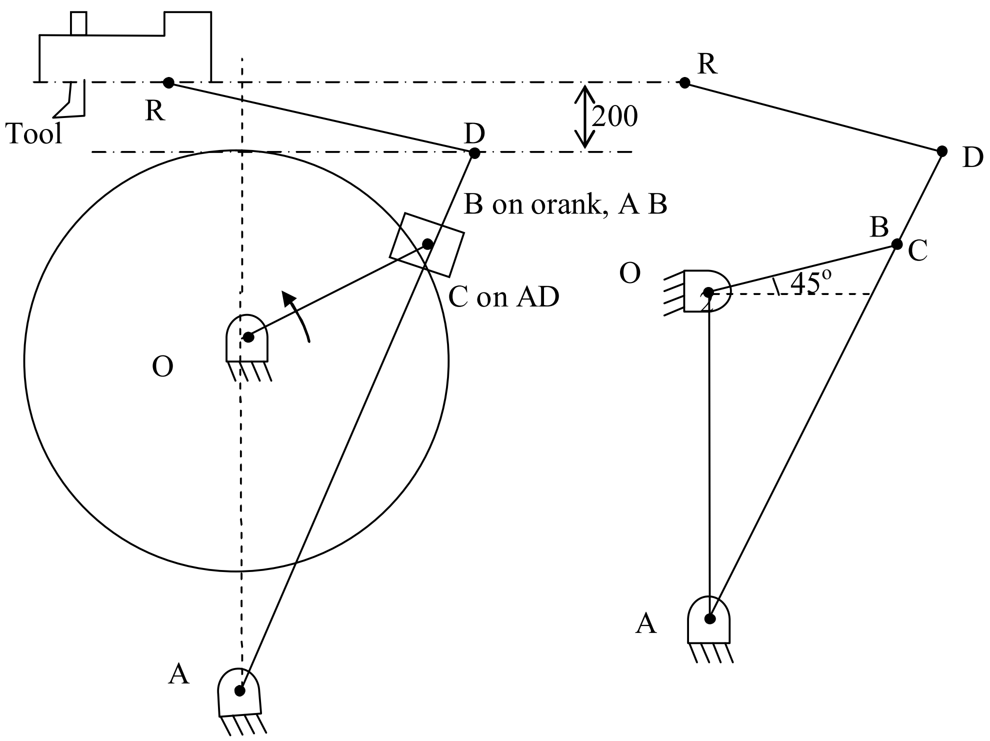
The crank O_1B makes an angle of 45^o with the vertical and rotates at 40 rpm in the CCW direction. Find:

iii) Acceleration of the Ram R, velocity of cutting tool, and

iv) Angular Acceleration of link AD.

Solution:

<u>Step 1</u>: Draw the configuration diagram.



Step 2: Determine velocity of point B.

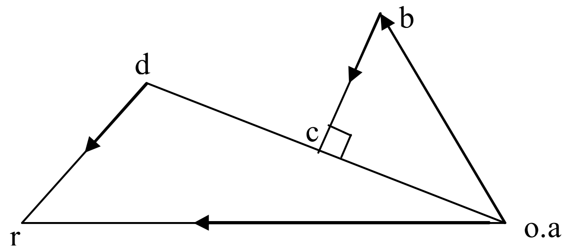
$$V_b = \omega_{OB} \times OB$$

$$\omega_{OB} = \frac{2\pi N_{O1B}}{60} = \frac{2\pi \times 40}{60} = 4.18\,rad/sec$$

$$V_b = 4.18 \times 0.3 = 1.254\ m/sec$$

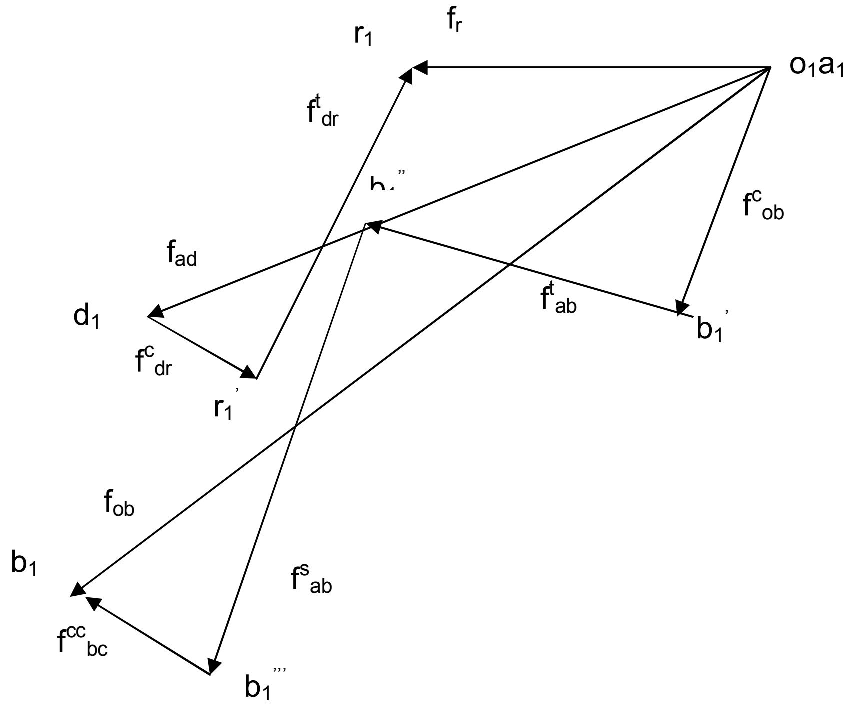
Step 3: Draw velocity vector diagram.

Choose a suitable scale 1 cm = 0.3 m/sec



Step 4: prepare table showing the acceleration components

Sl. No.	Link	Magnitude m/s^2	Direction	Sense
1.	OB	$f^c_{ob} = \omega^2 r = 5.24$	Parallel to OB	→ O –
2.	AC	$f^c_{ac} = \omega^2 r$ $f^t_{ac} = \alpha r$	Parallel to AB $\perp^r$ to AB	→ A –
3.	BC	$f^s_{bc} = \alpha r$ $f^{cc}_{bc} = 2v\omega =$	Parallel to AB $\perp^r$ to AC	– –
4.	DR	$f^c_{bd} = \omega^2 r = 20$ $f^t_{bd} = \alpha\ r$	Parallel to DR $\perp^r$ to BD	→ D –
5.	Slider R	$f^t_{bd} = \alpha r$	Parallel to slider motion	–



Acceleration of Ram = fr = o_1 r

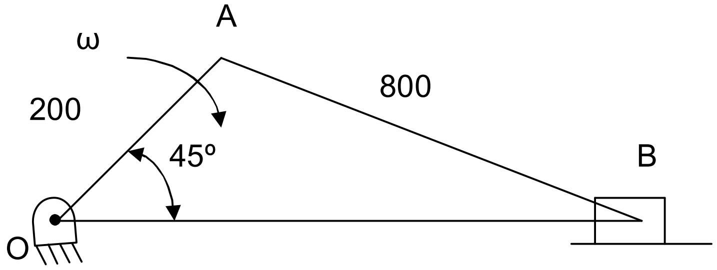
Angular Acceleration of link AD

$$\alpha_{bd} = \frac{f_{bd}}{BD}$$

KLENIN'S Construction

This method helps us to draw the velocity and acceleration diagrams on the construction diagram itself. The crank of the configuration diagram represents the velocity and acceleration line of the moving end (crank).

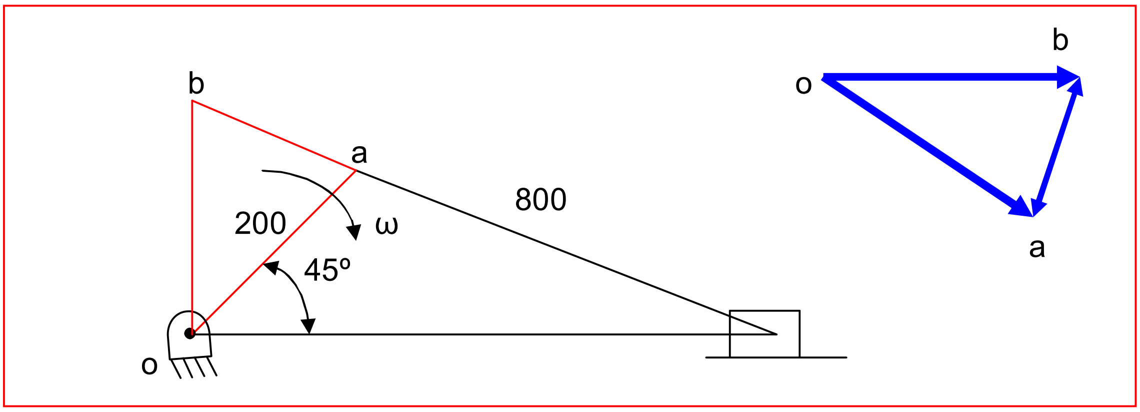
The procedure is given below for a slider crank mechanism.



To draw the velocity vector diagram:

Link OA represents the velocity vector of A with respect to O.

V_{oa} = oa = ω r = ω OA.



Draw a line perpendicular at O, extend the line BA to meet this perpendicular line at b. oab is the velocity vector diagram rotated through 90º opposite to the rotation of the crank.

Acceleration diagram:
The line representing Crank OA represents the acceleration of A with respect to O. To draw the acceleration diagram follow the steps given below.

- Draw a circle with OA as radius and A as centre.
- Draw another circle with AB as diameter.
- The two circles intersect each other at two points C and D.
- Join C and D to meet OB at b_1 and AB at E.

O_1, a_1, b_{a1} and b_1 is the required acceleration diagram rotated through 180º.